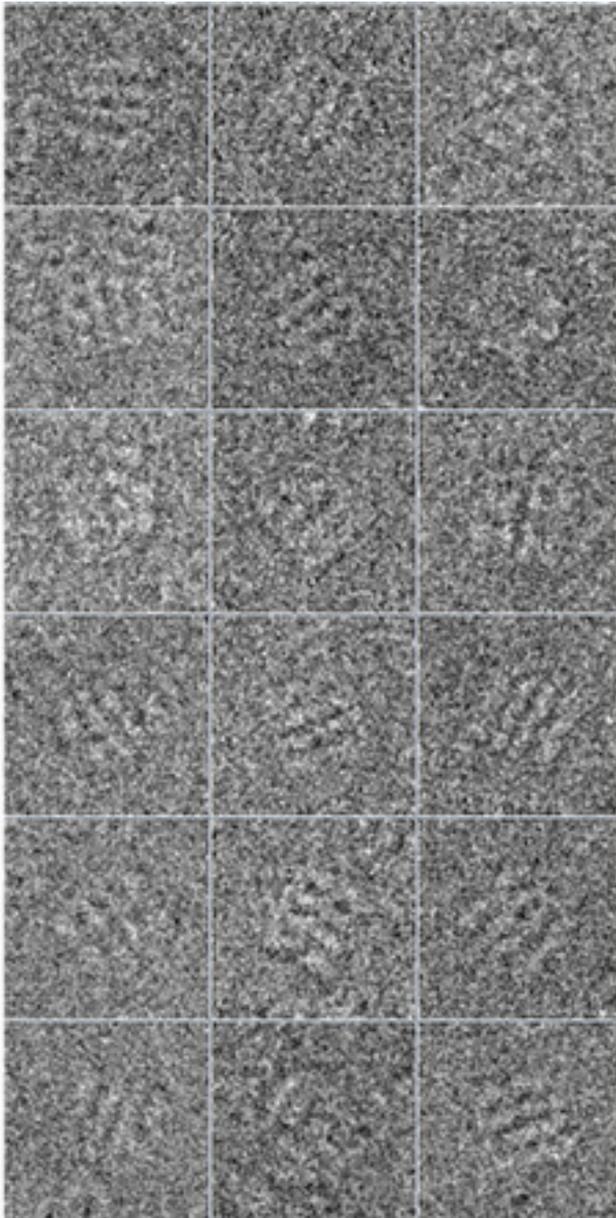


# From 2D particles to 3D reconstruction

Hélène Malet  
Renafobis 2017

**We have collected your nice 2D images. But there are very noisy...**

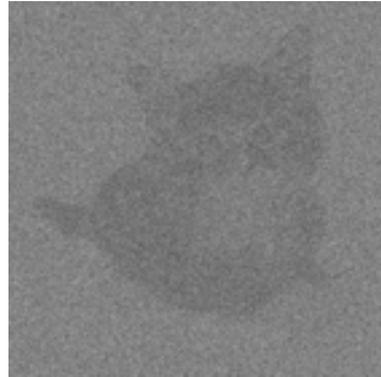


What would you do to improve signal/noise ?

**Make groups of similar particles and average them !**

**= 2D classification**

A single image



Average of several images with the same orientation

4

16

64

256

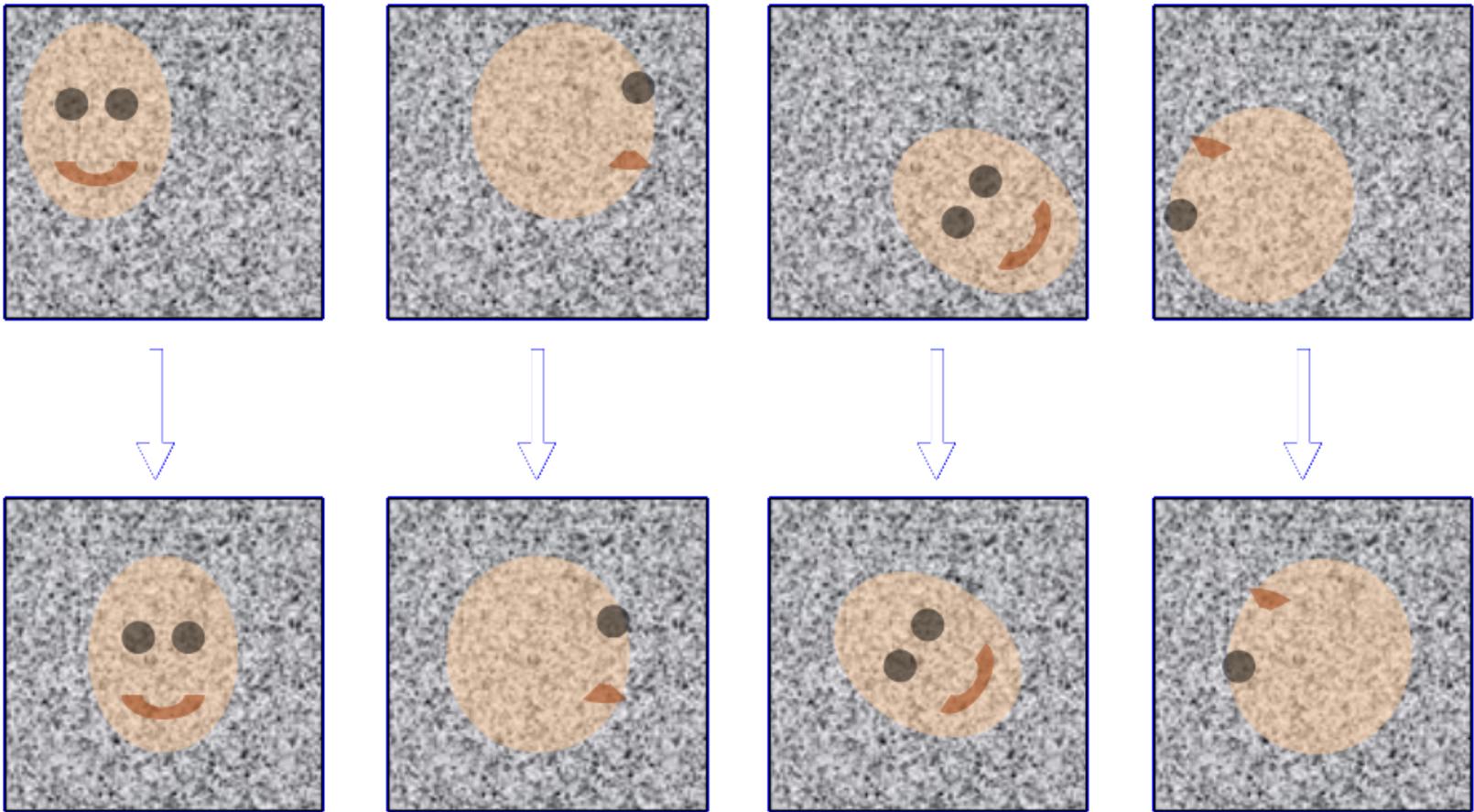
1024



The problem: there are several orientations so we need to separate into several class averages

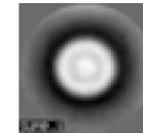
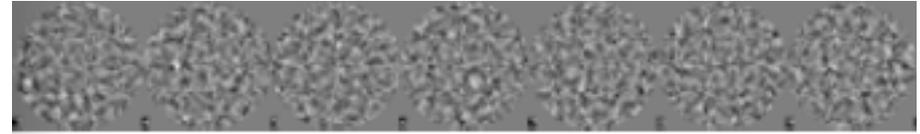
# How to create homogeneous 2D class averages?

- Translation of images to center them

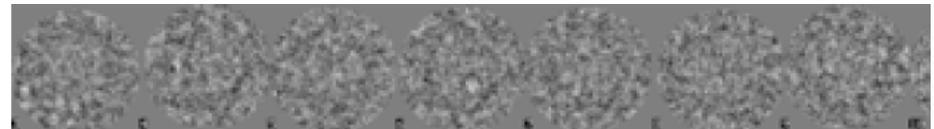
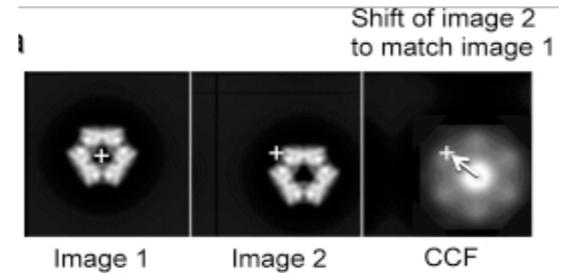
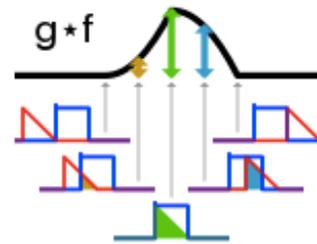
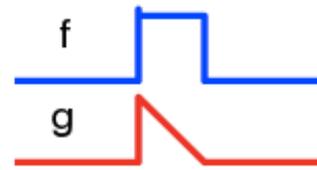


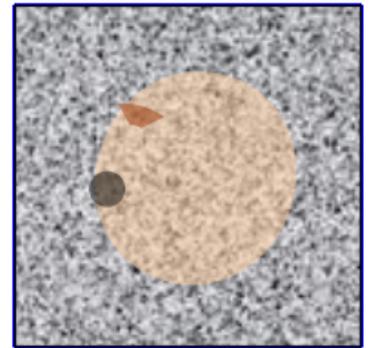
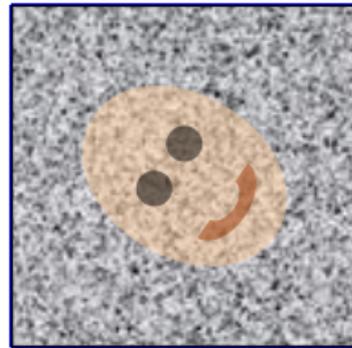
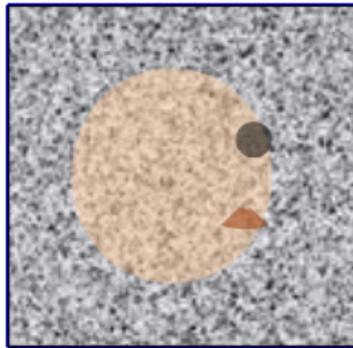
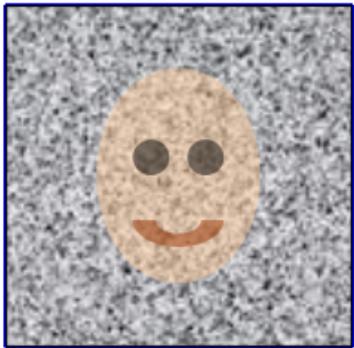
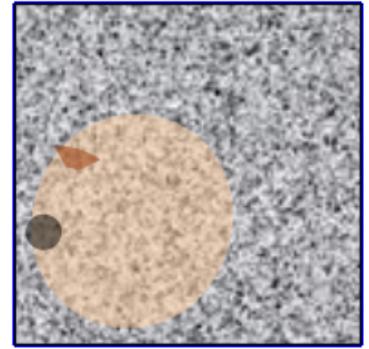
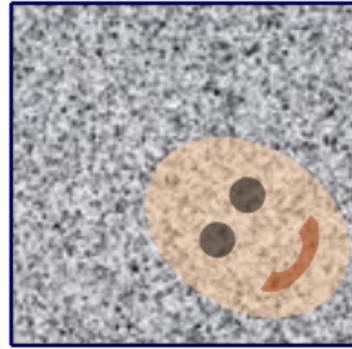
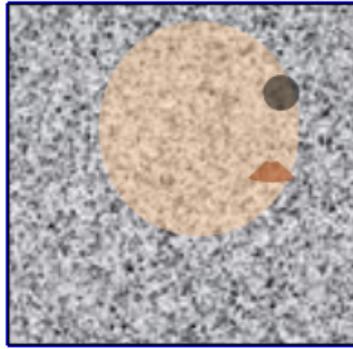
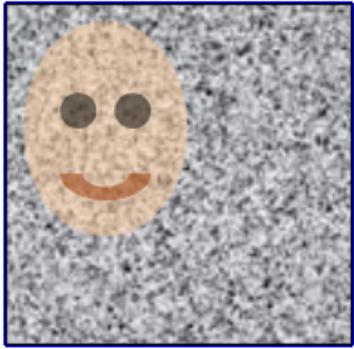
# Method used to center images

- Average of all images
- Rotational average
- Cross-correlation between each image and the rotational average
- Translation of each image to center it !

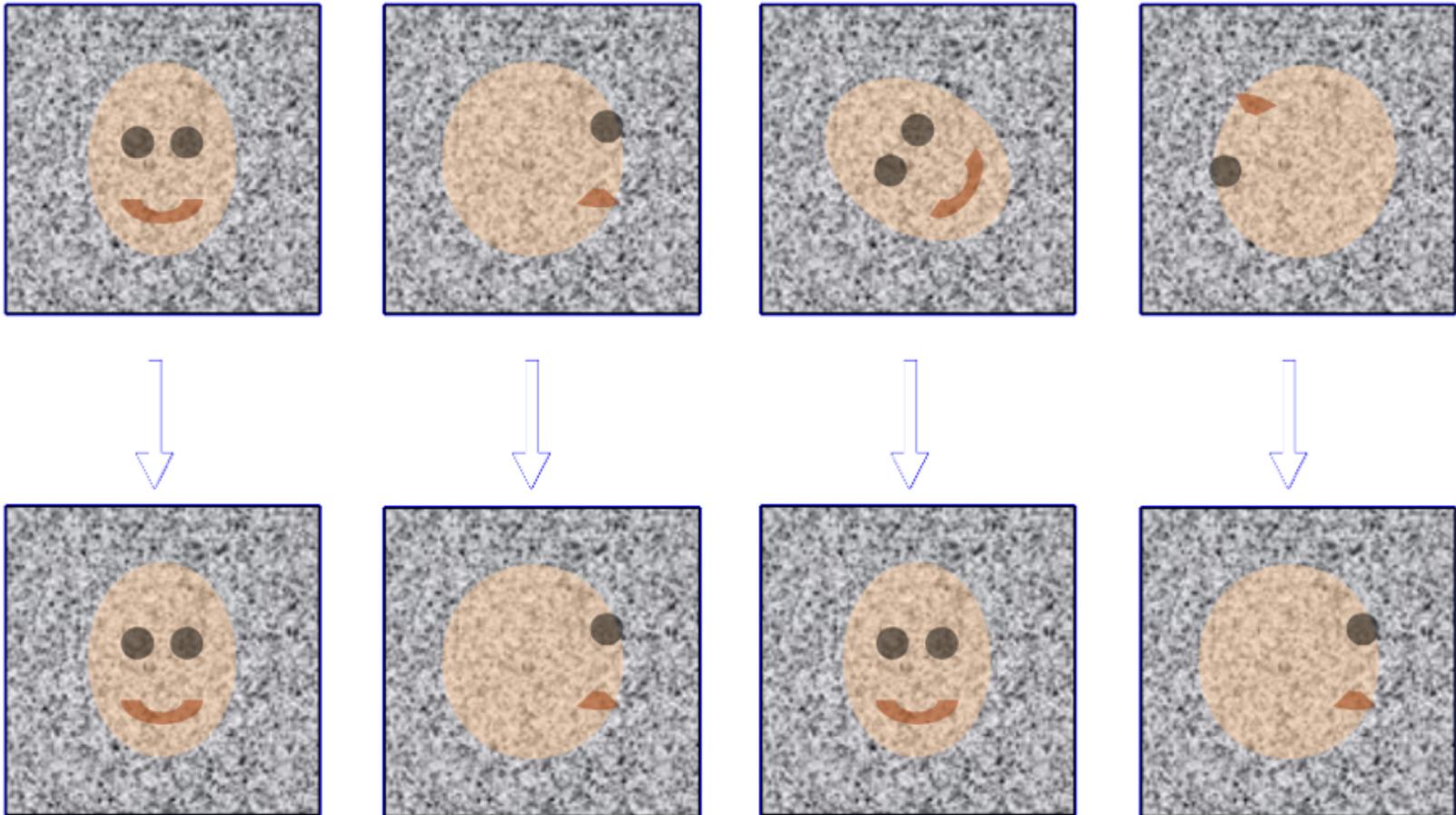


Cross-correlation

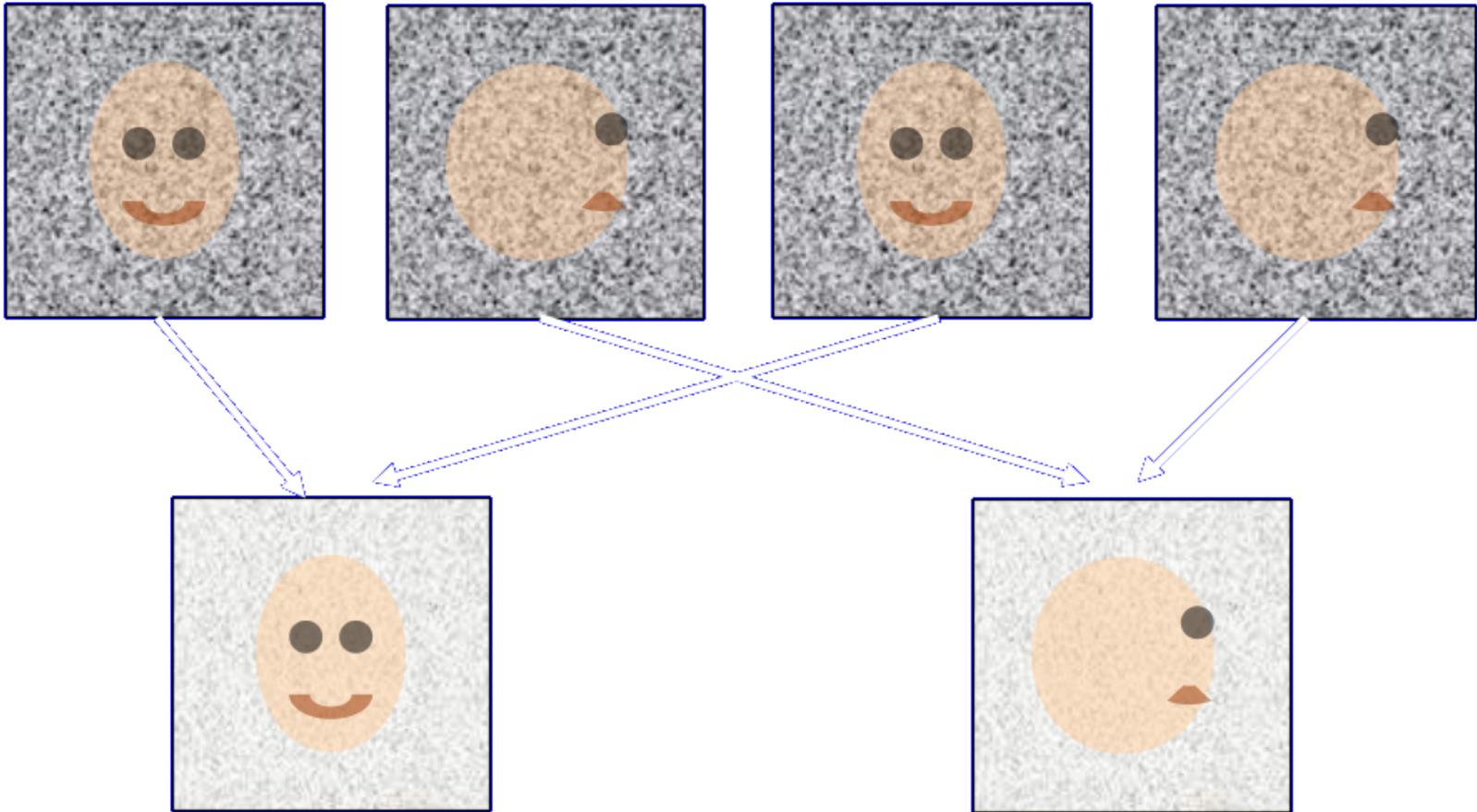




- Rotation (requires a reference). Again done using cross-correlations.



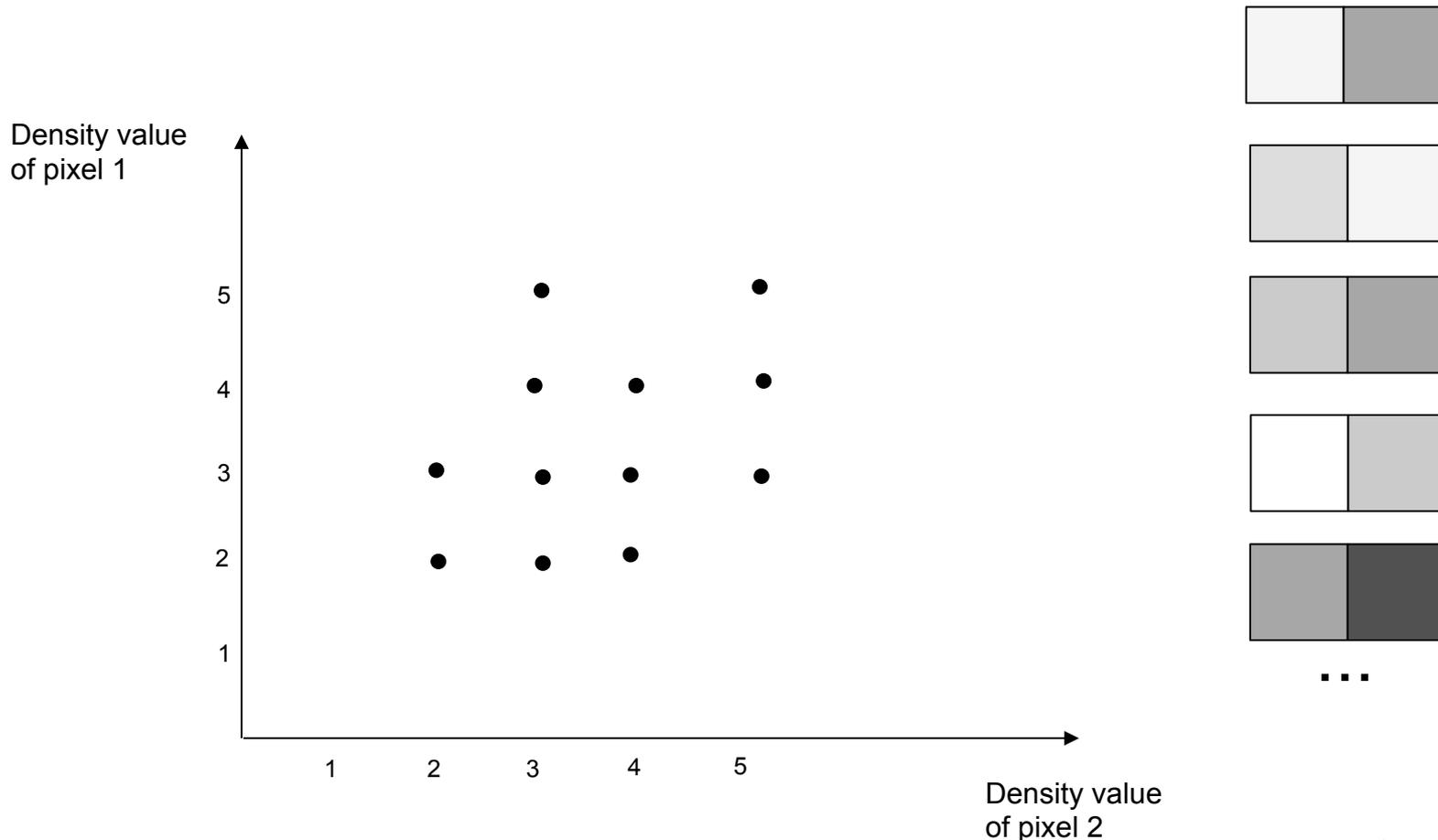
- Centered and rotated images with low signal to noise ratio



- Class averages with higher signal to noise.

# Principle of Multivariate Statistical Analysis to obtain 2D class average

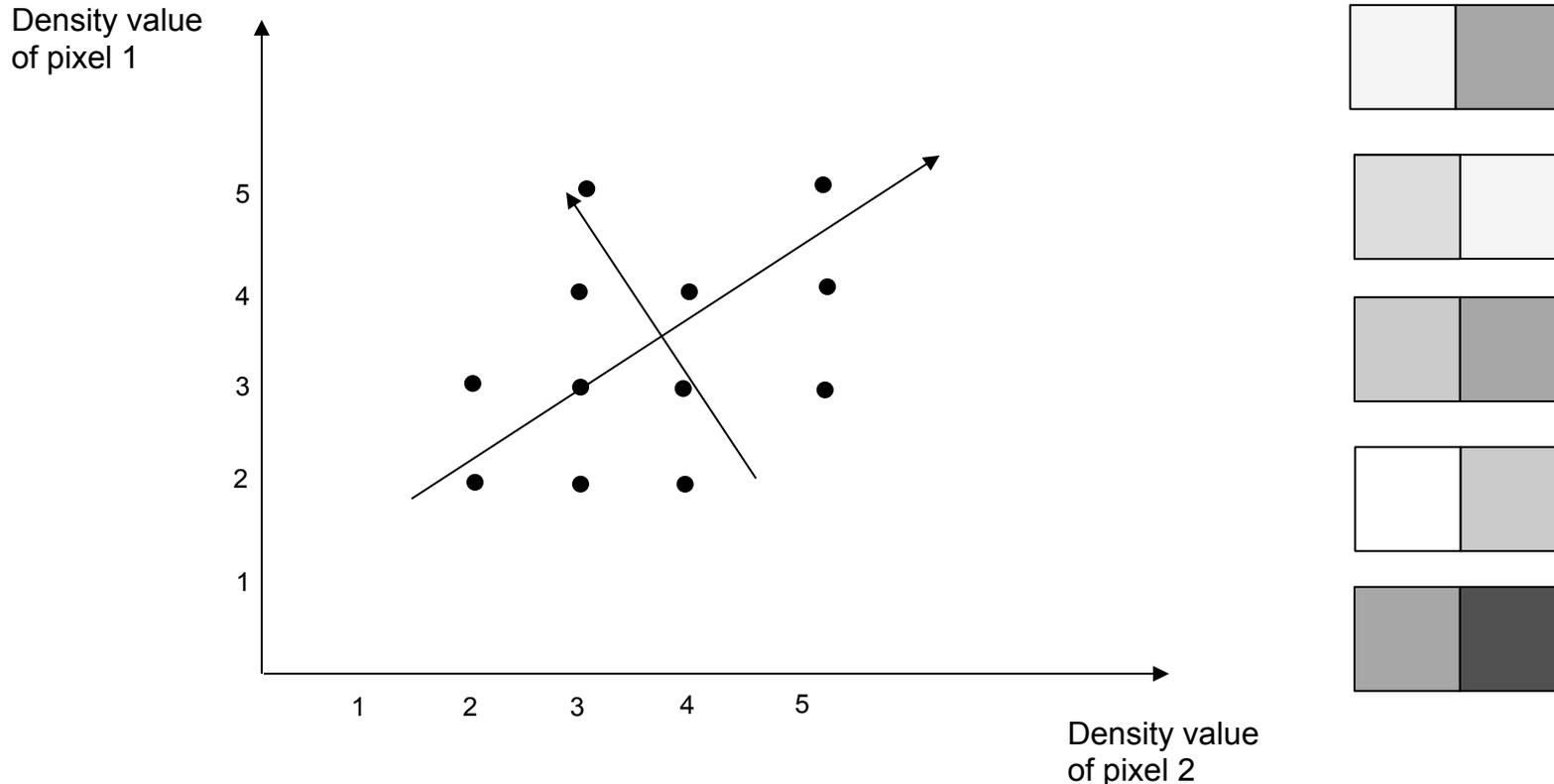
- Let's consider the simplest case: images with only 2 pixels



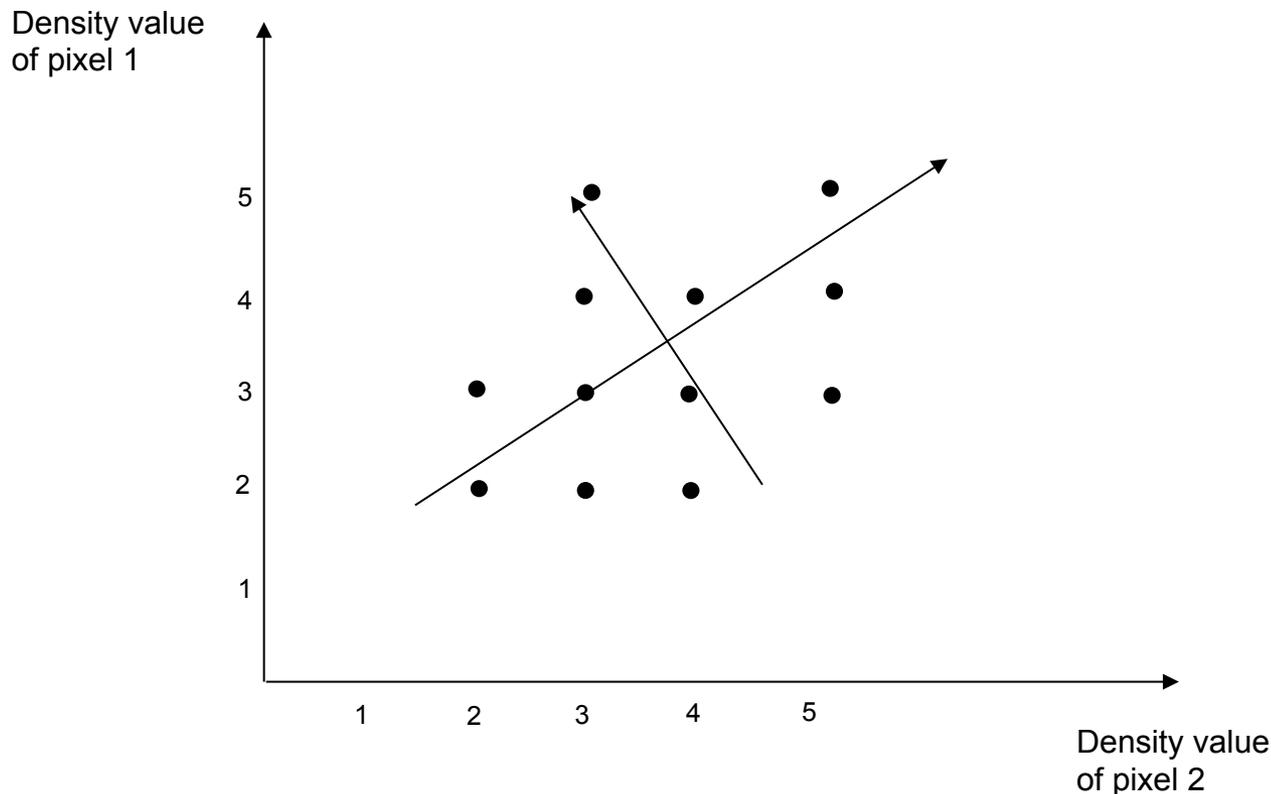
*Adapted from B. Klaholz*

# Aim of the MSA: adapt the coordinate system to the shape of the data cloud

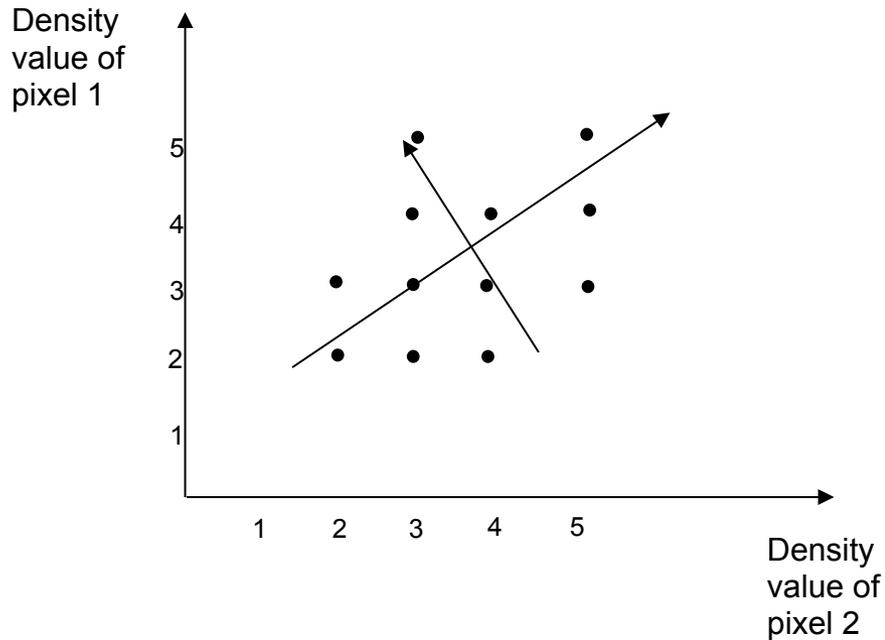
- 1st axis: longest elongation of the data cloud, i.e. highest variance
- 2st (orthogonal) axis: corresponds to the next strongest variance



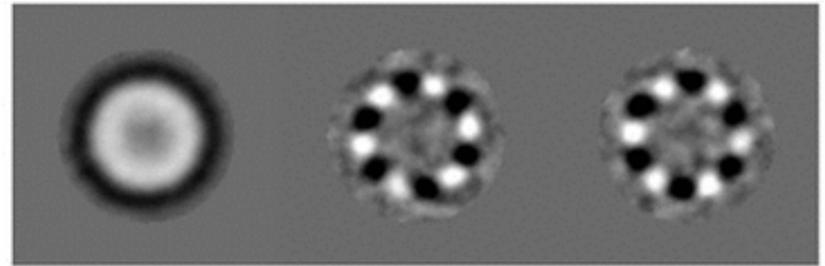
- If we now have images with 100x100 pixels. Instead of having two axes, we will have 100x100 axes....
- But we can still rotate the coordinate systems to make it correspond to the largest elongations of the data cloud.
- Only few axes = eigenvectors will correspond to the main directions of variations. This new coordinate system will be used for an MSA analysis → data reduction



- Eigenimages will be calculated and will correspond:
  - for the 1st one to the total sum
  - for the others: to the highest variances of the dataset

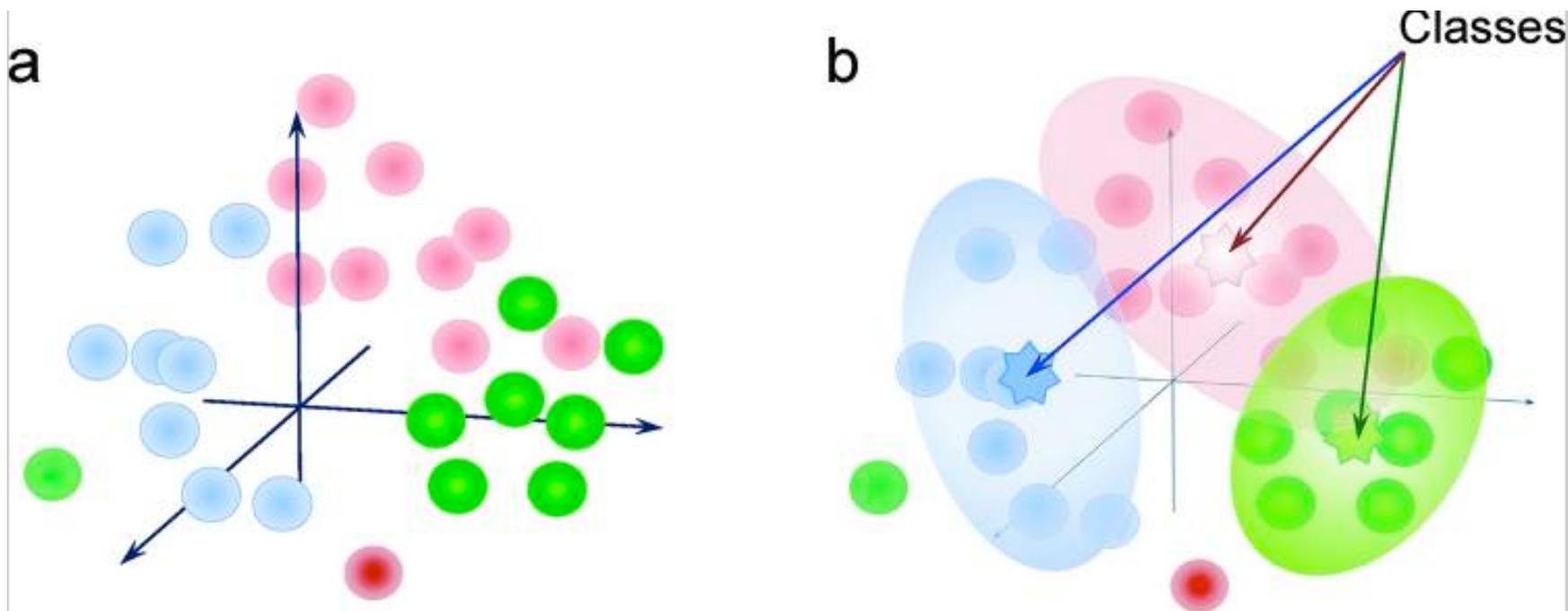


Eigenimages

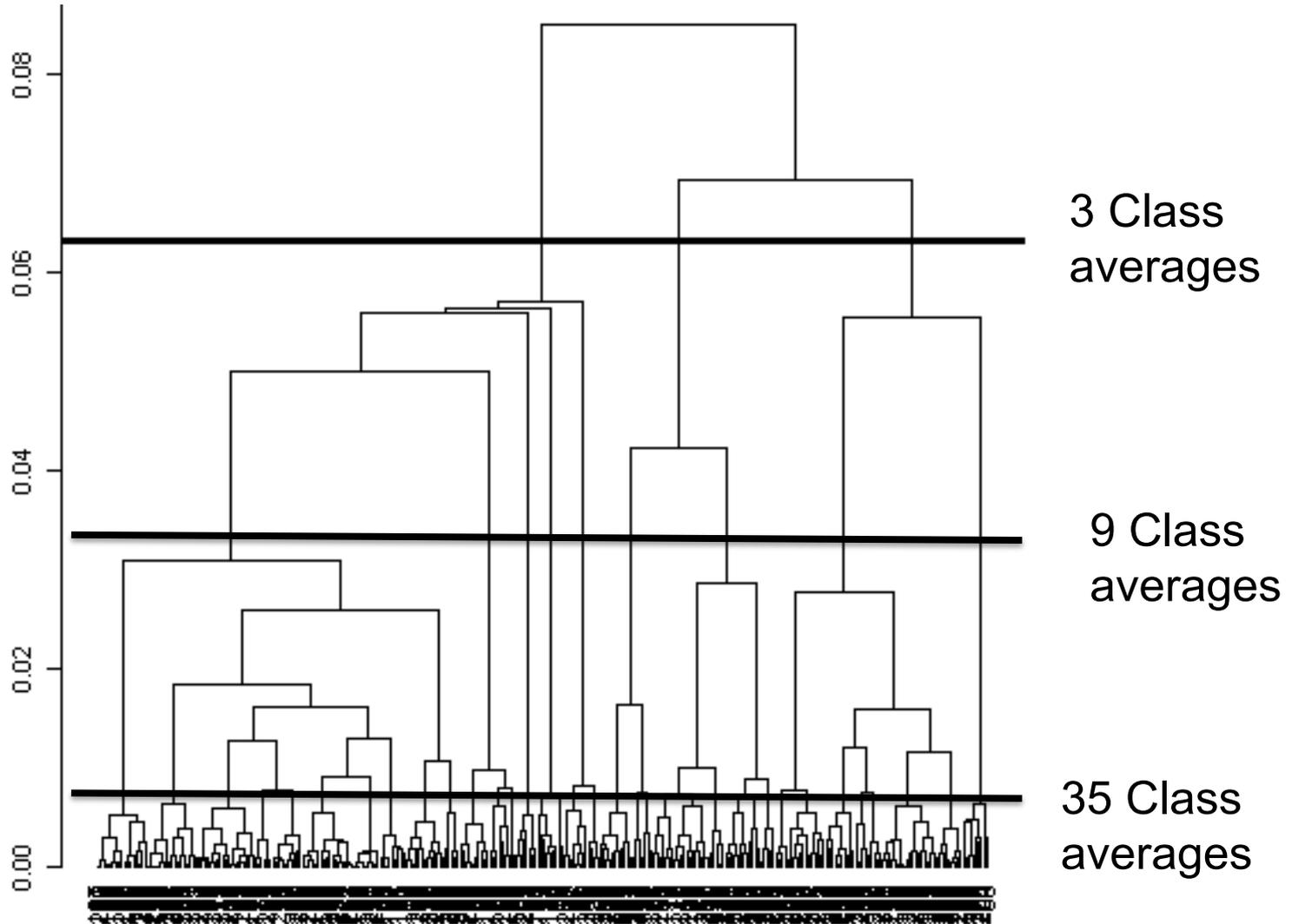


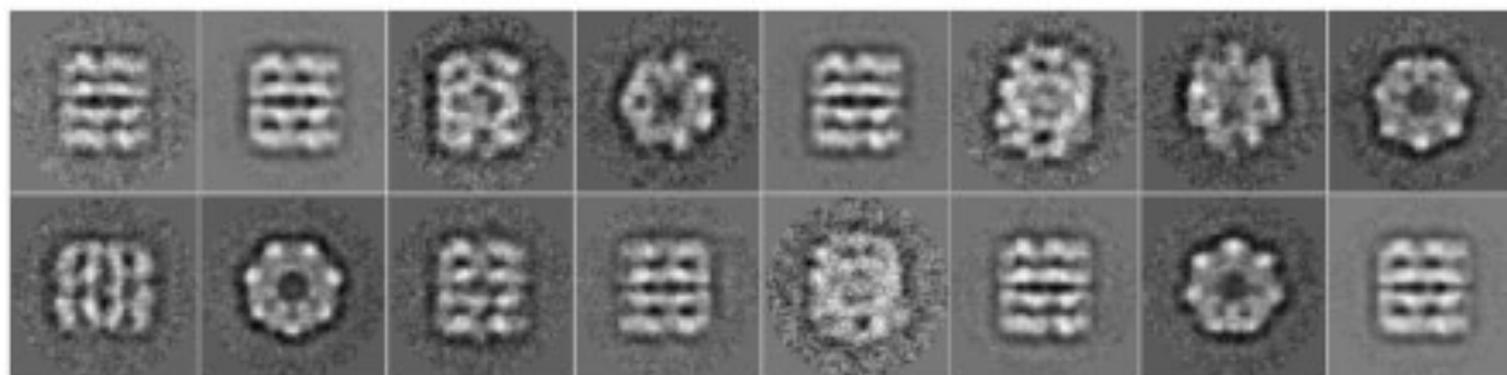
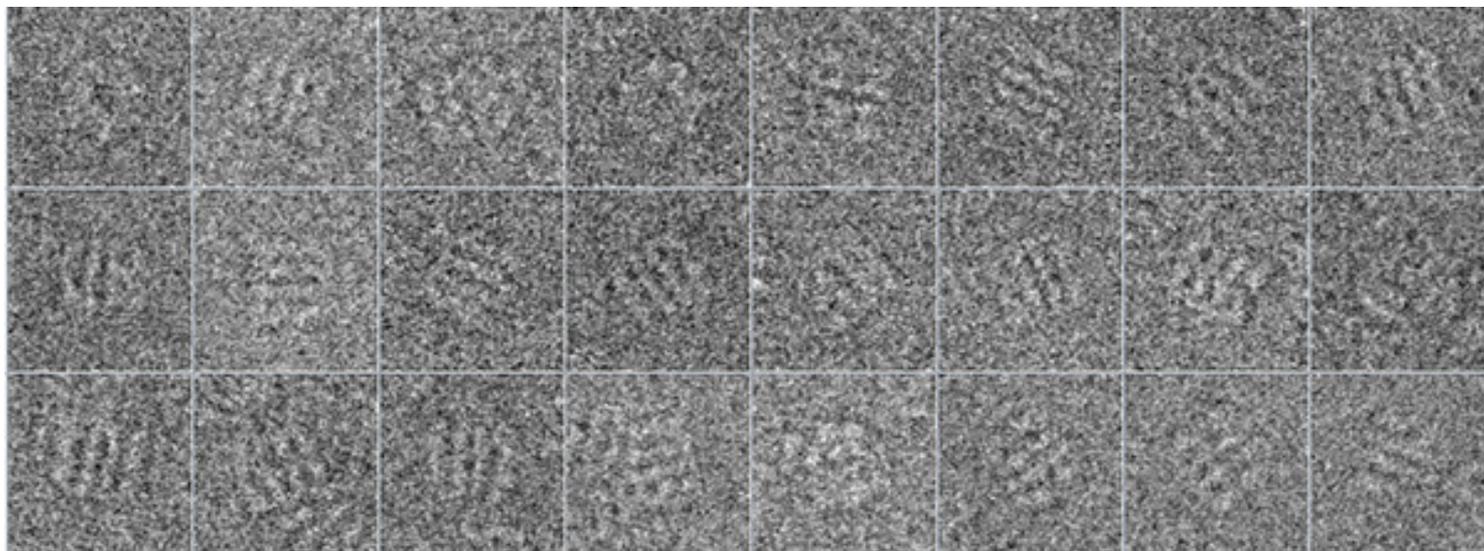
# Clustering

- minimization of the intraclass variance in a cluster (between the members of the cluster)
- maximization of the interclass variance between the centers of mass of the clusters

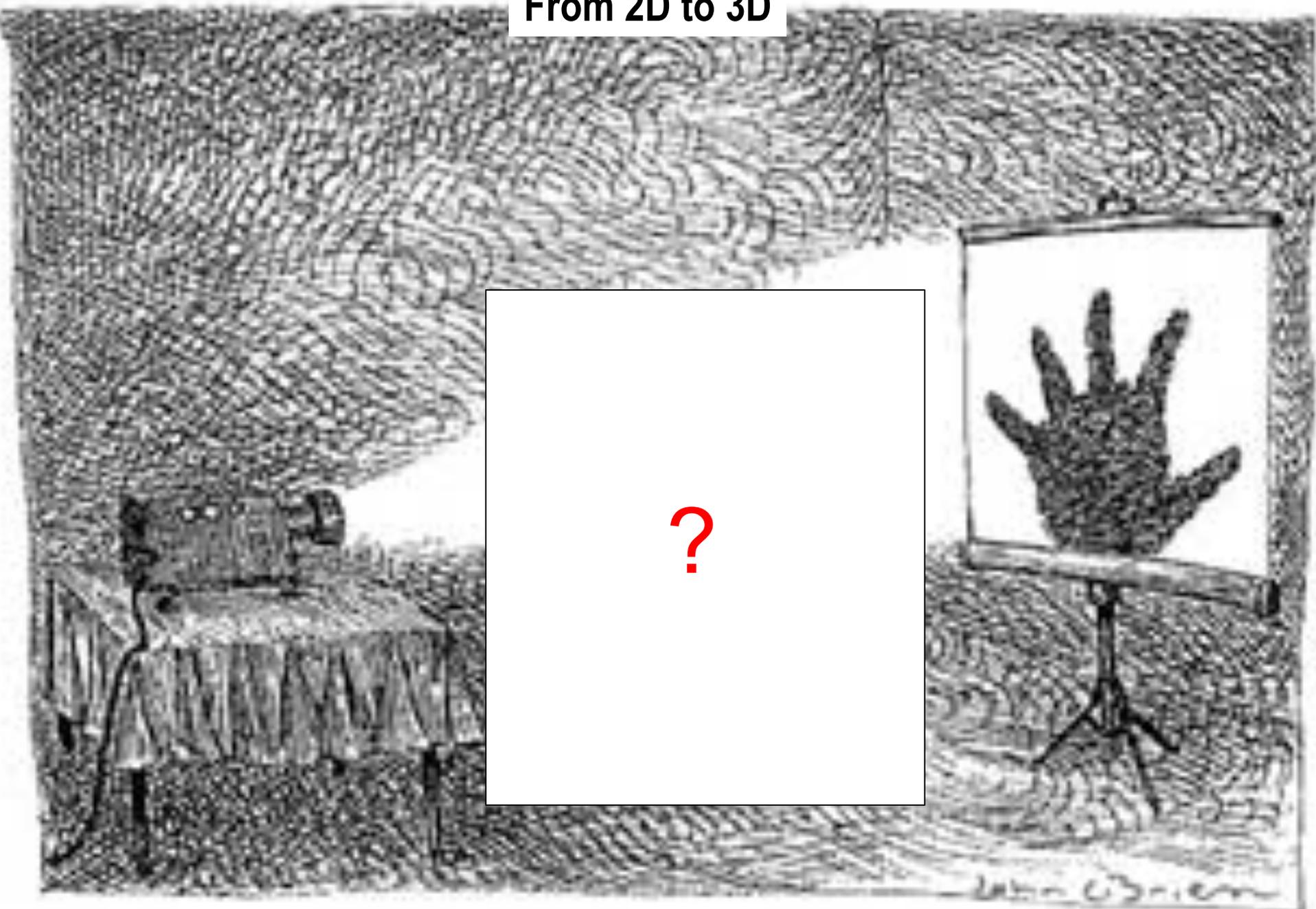


# Hierarchical (agglomerative) ascendant classification





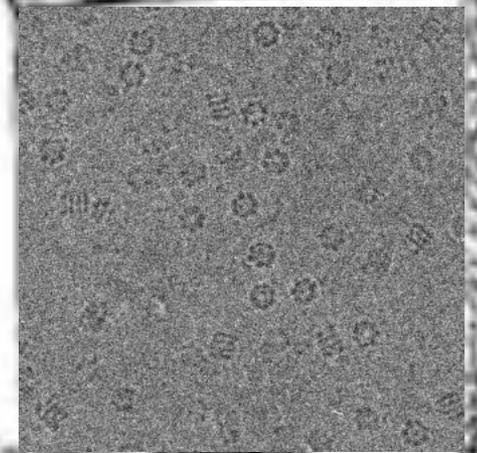
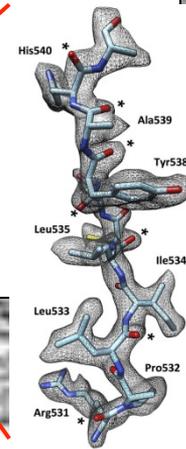
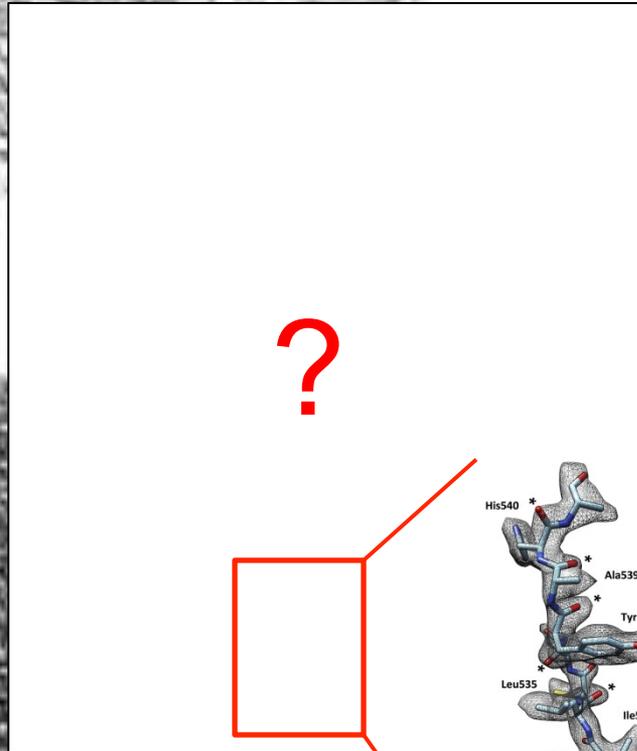
# From 2D to 3D



# From 2D to 3D



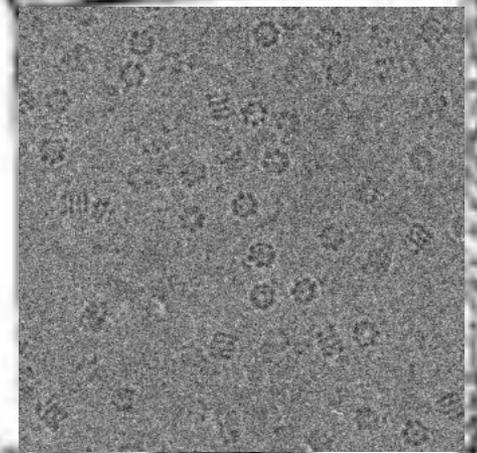
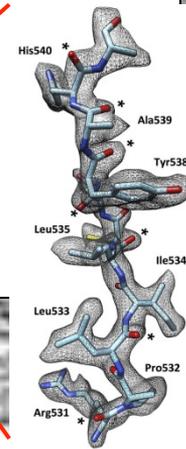
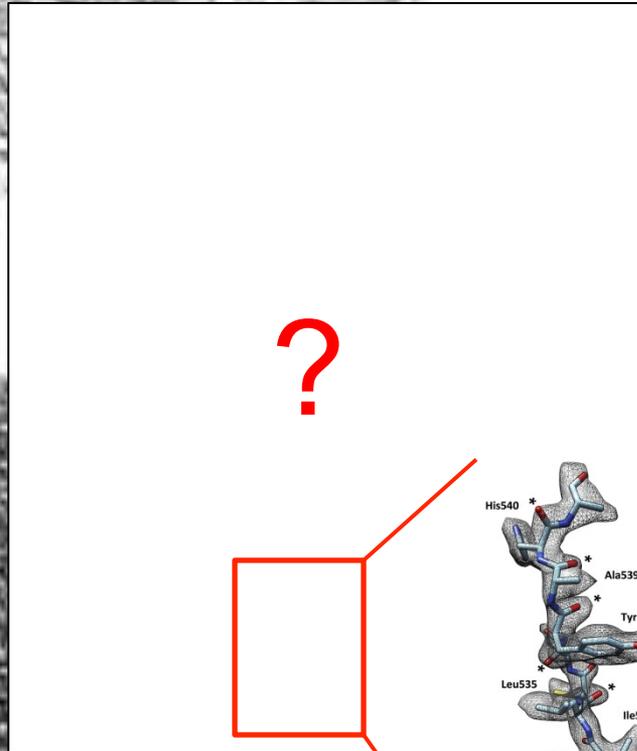
Electron microscope



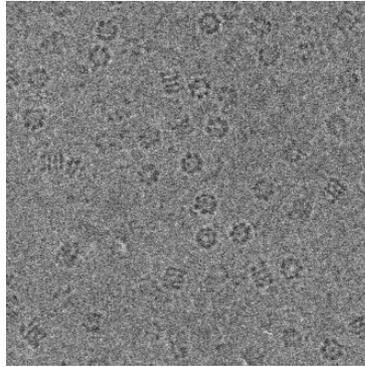
# From 2D to 3D



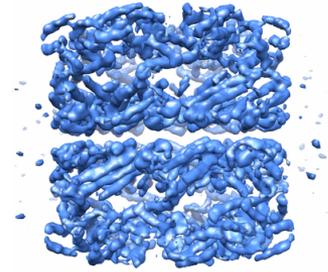
Electron microscope



**From 2D...**



**... To 3D**



- Step 1: determine orientations



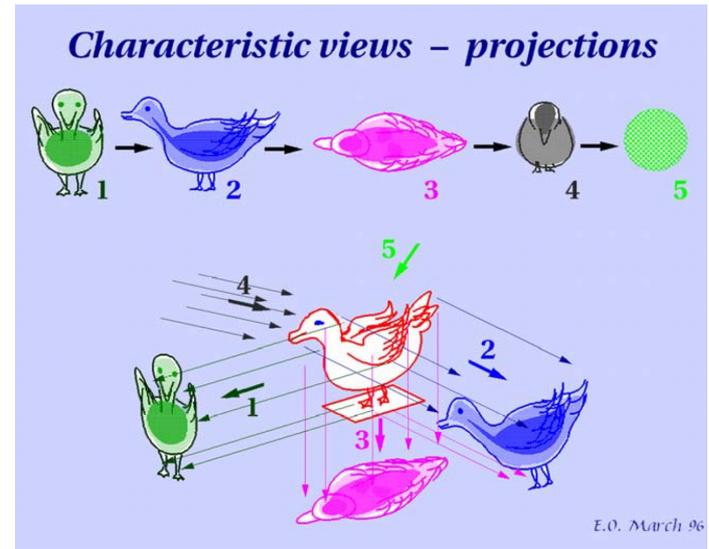
- Step 2: combine these orientations to obtain a 3D reconstruction



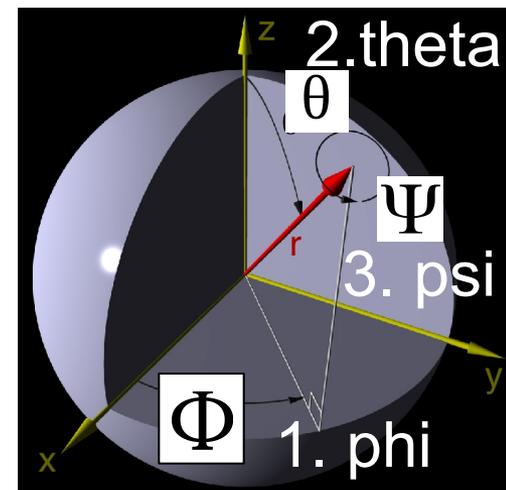
- Step 3: refine the structure

# Step 1: How to determine orientations?

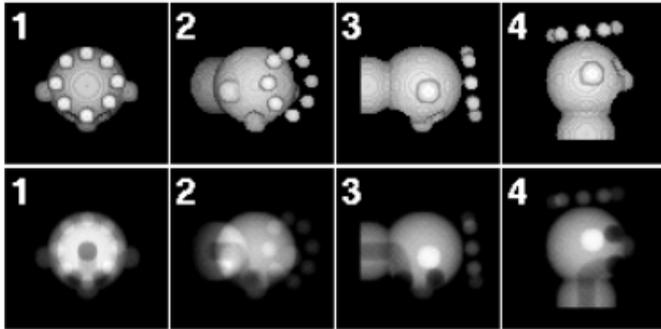
- We need to assign to which view which image corresponds.



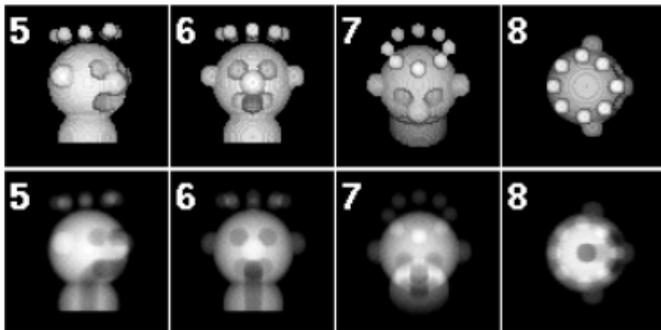
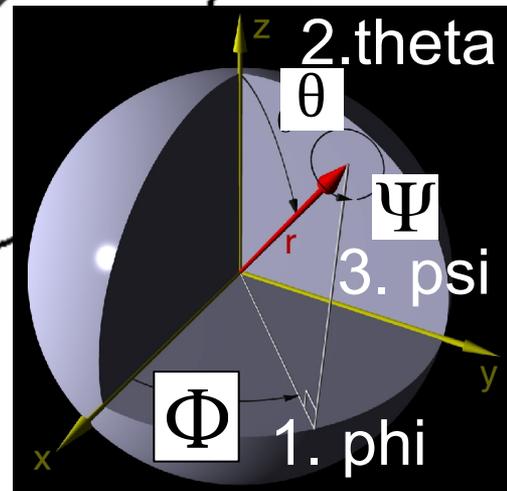
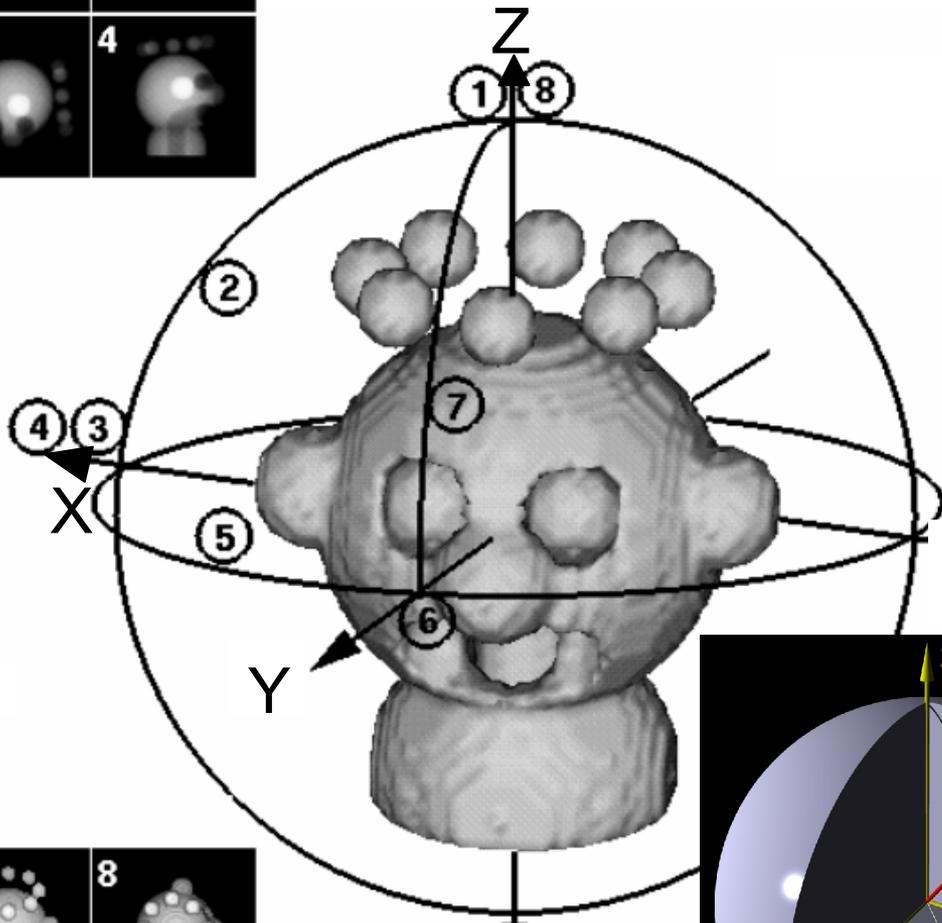
- We need to assign euler angles to images !



# Euler angles



	Phi	Theta	Psi
	$\varphi$	$\theta$	$\psi$
1:			
2:			
3:			
4:			
5:			
6:			
7:			
8:			



# Step 1: How to determine orientations?

## 1st method: with 2D class averages

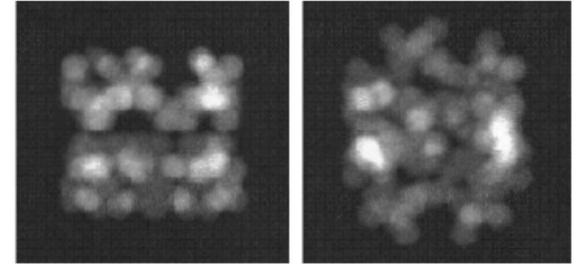
### Angular reconstitution

- Determination of the euler angles of the projection: computationally, using the common line theory

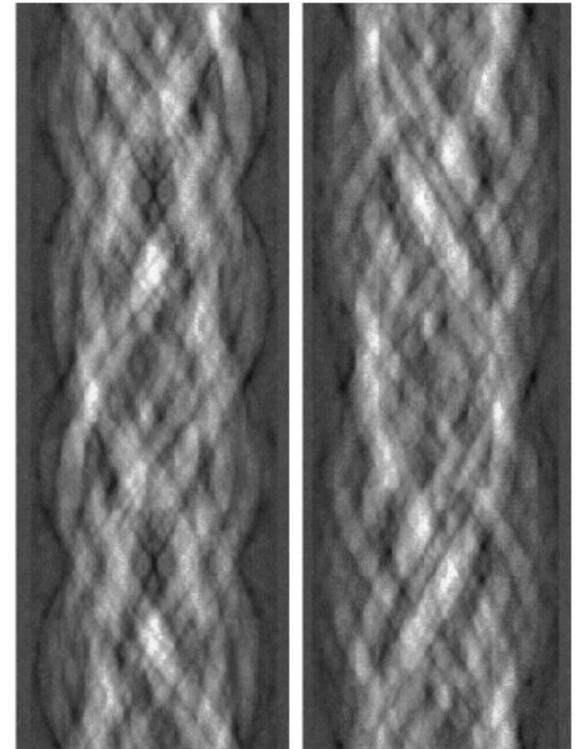
“two projections of the same object share a common line”

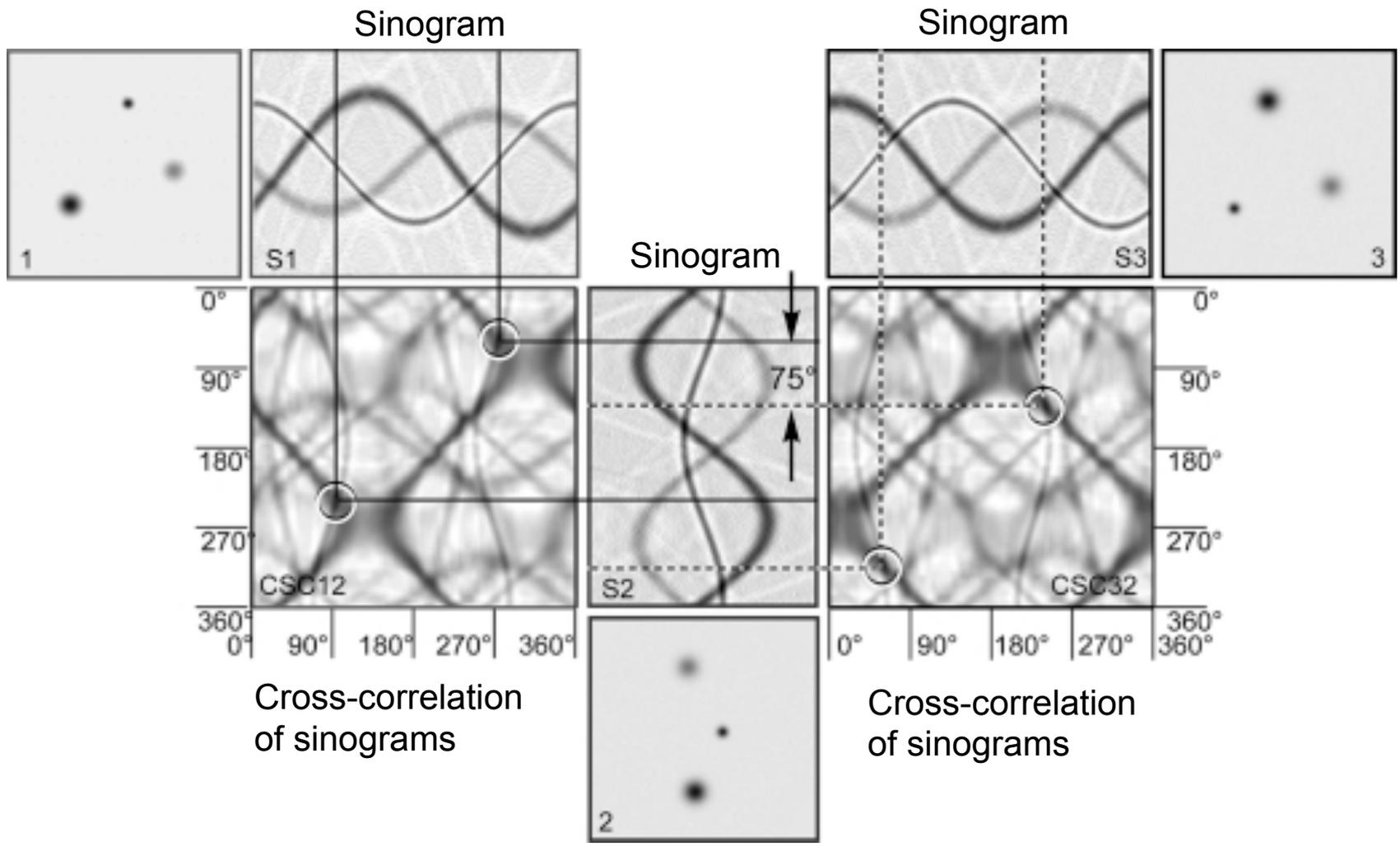
- Done on 2D class averages
- Necessity to be centered

Projections



Sinograms



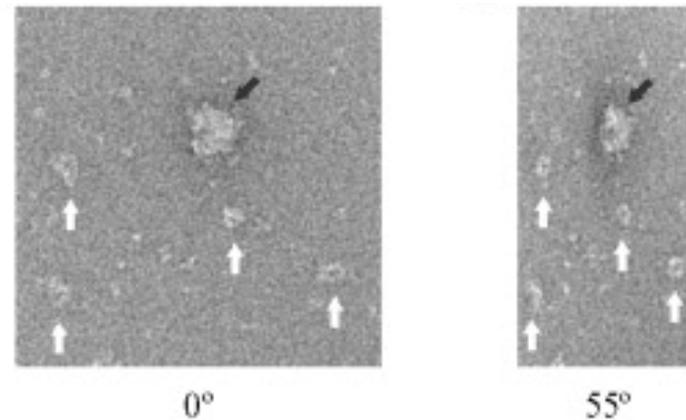
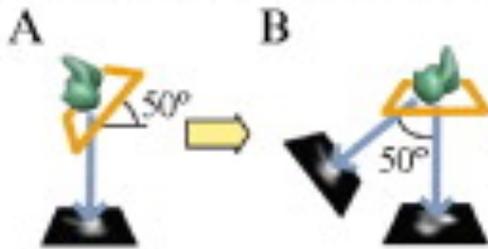


# Step 1: How to determine orientations?

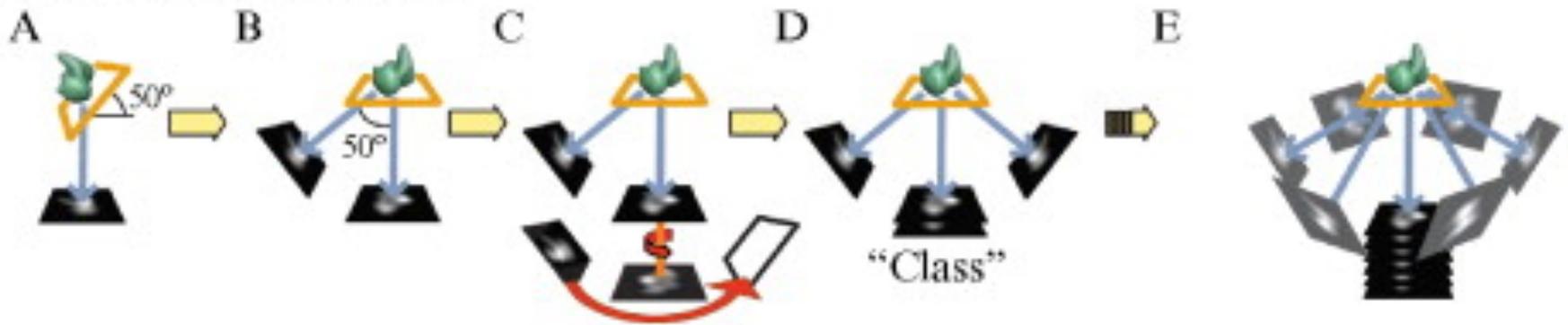
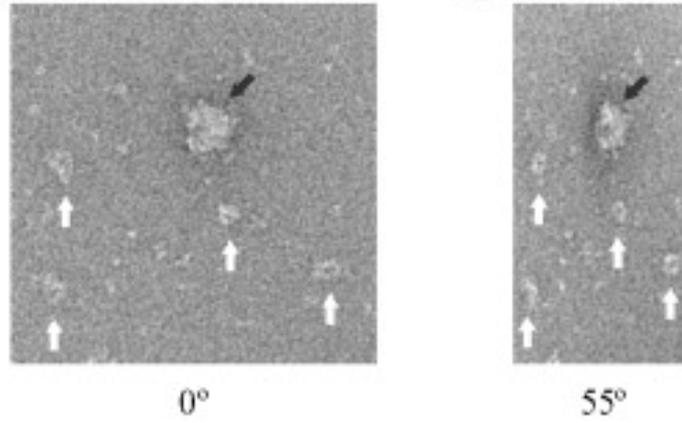
## 2nd method: with data collection of tilt pairs

### Random conical tilt

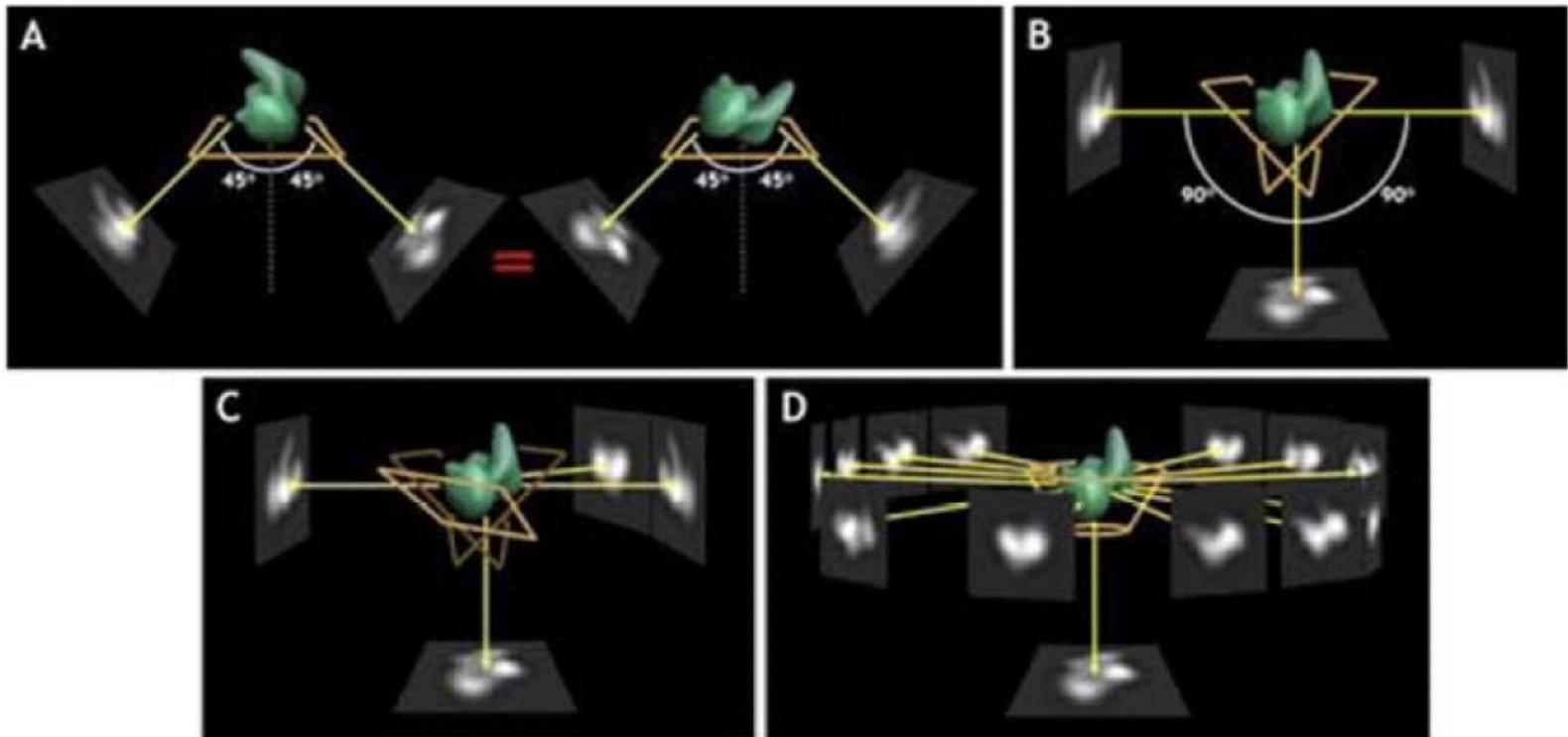
- From projection pairs (usually 0, 60 degree)



- Possible to assign relative angles between images



- Can also be done with images at -45 and 45 degree
- Orthogonal random conical tilt

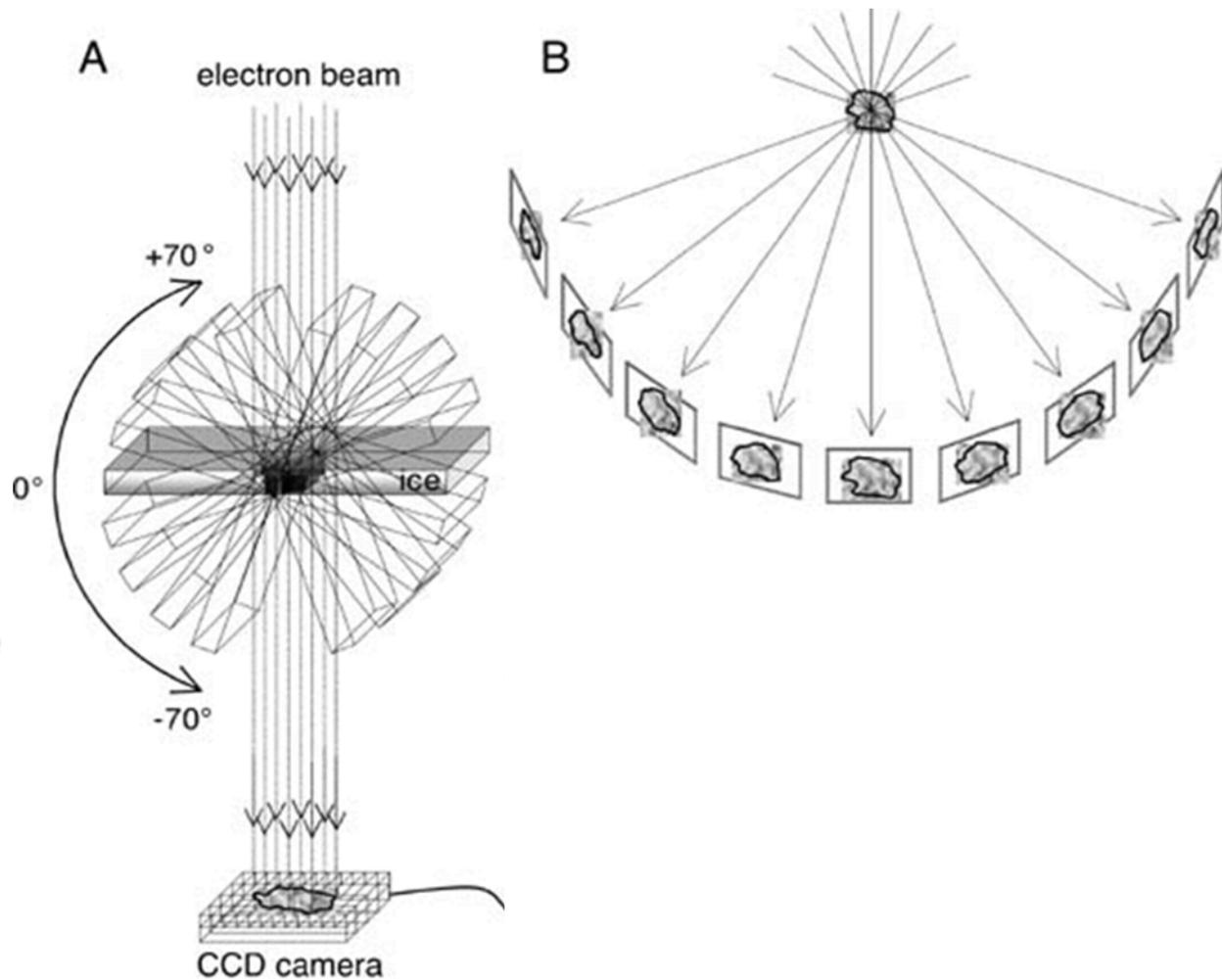


## Step 1: How to determine orientations?

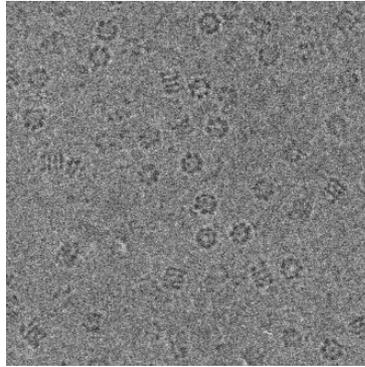
**3rd method: tomography**



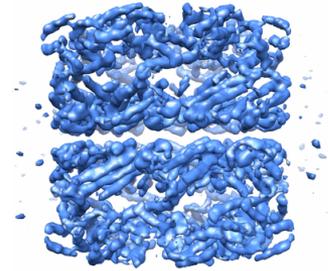
- Advantages:
  - Experimental determination
  - No classification
- Disadvantages:
  - Dose issues
  - Artefacts (we'll see later why...)



**From 2D...**



**... To 3D**



- Step 1: determine orientations

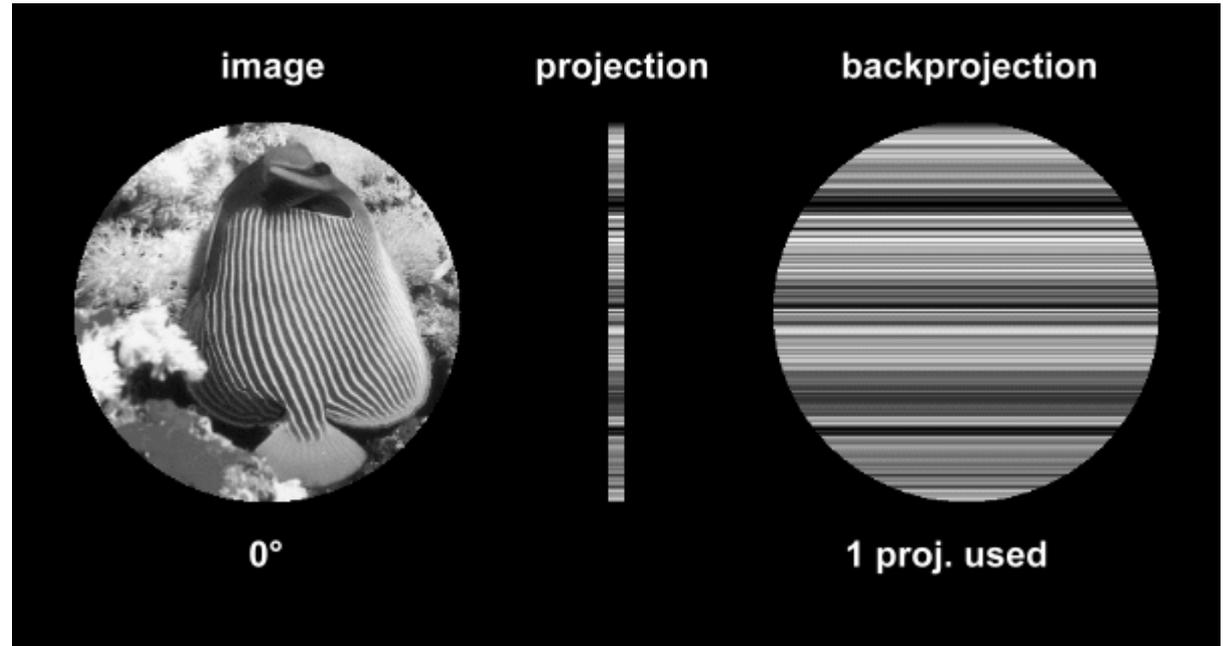
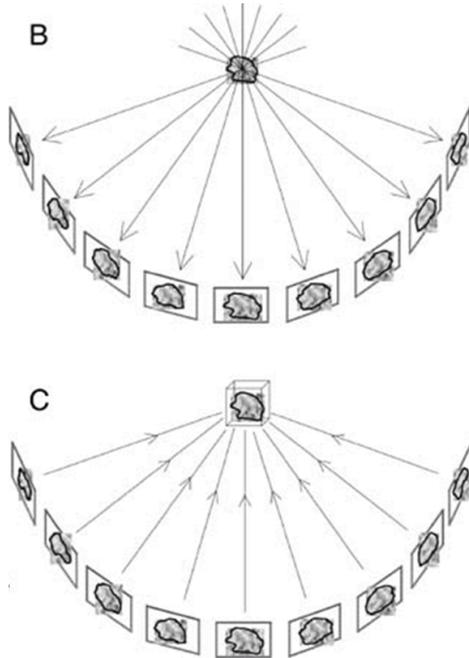


- Step 2: combine these orientations to obtain a 3D reconstruction

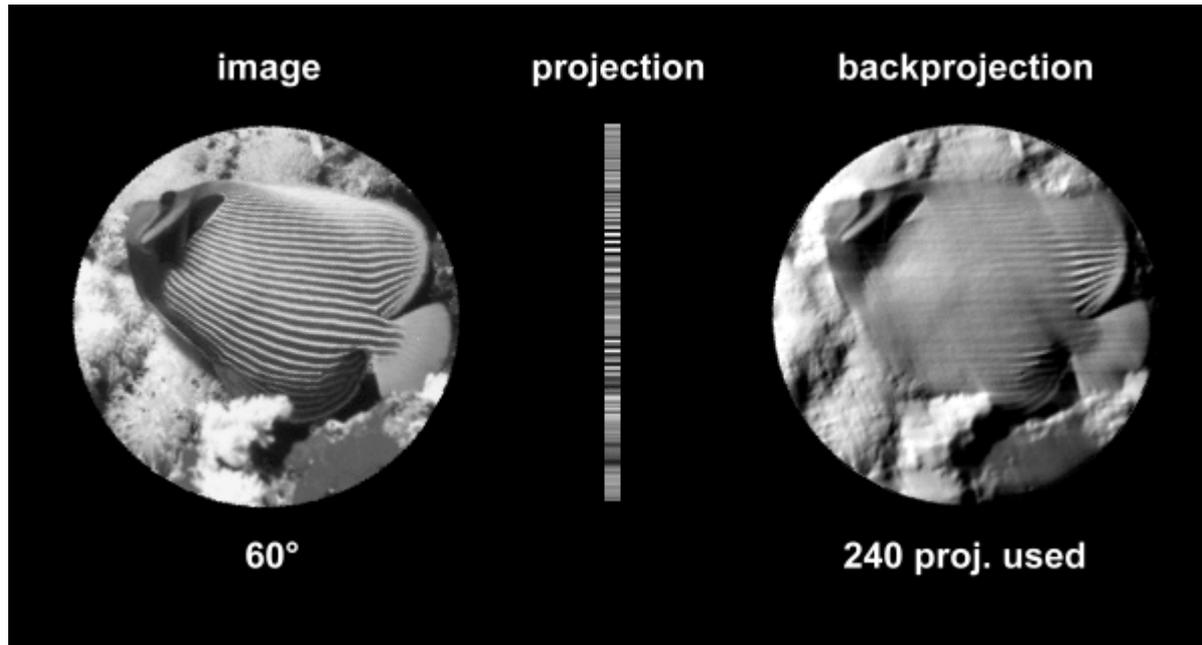


- Step 3: refine the structure

## Step 2: How to combine these orientations to obtain a 3D reconstruction?

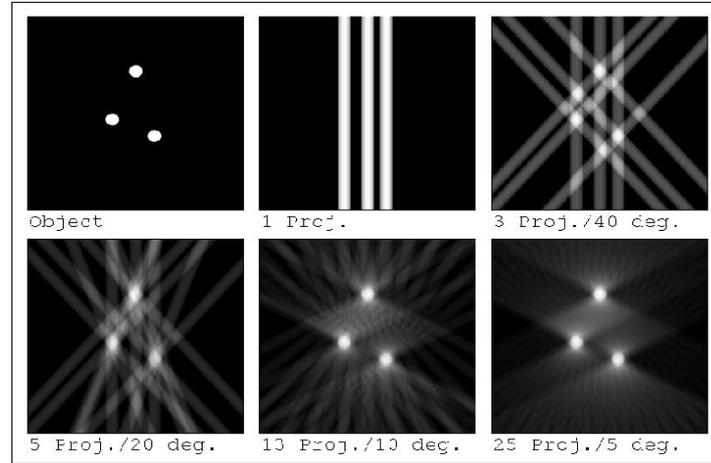


- **By backprojection !**
- One image : not sufficient. But if we know views from 0 to 60 degrees each  $\frac{1}{4}$  of degree ?

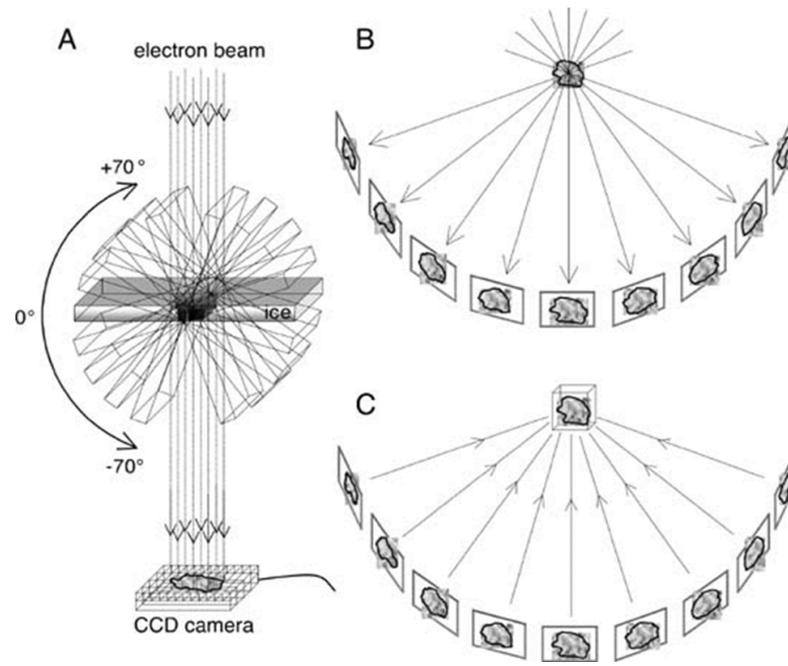


- Much better ! But still distorted. We should go to 90 degree to see no distortion

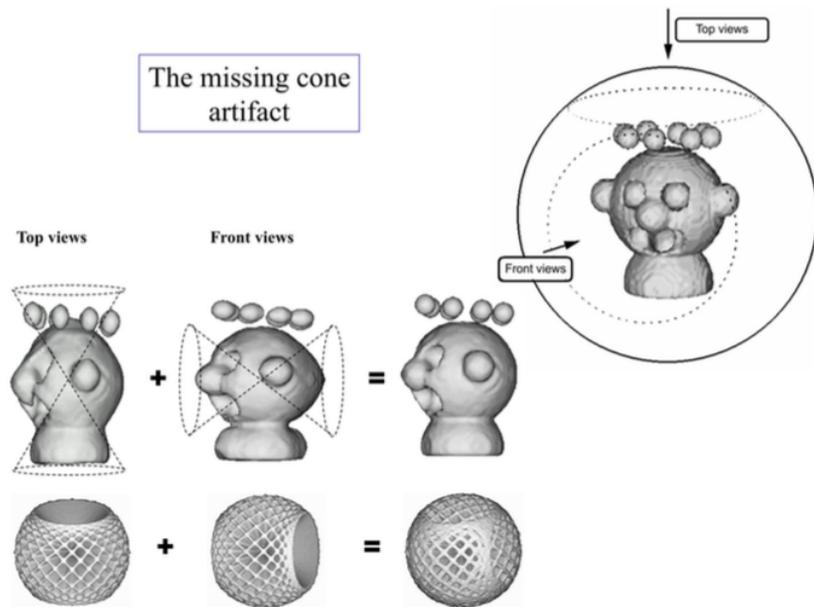
- Do we encounter such a problem of missing angles?



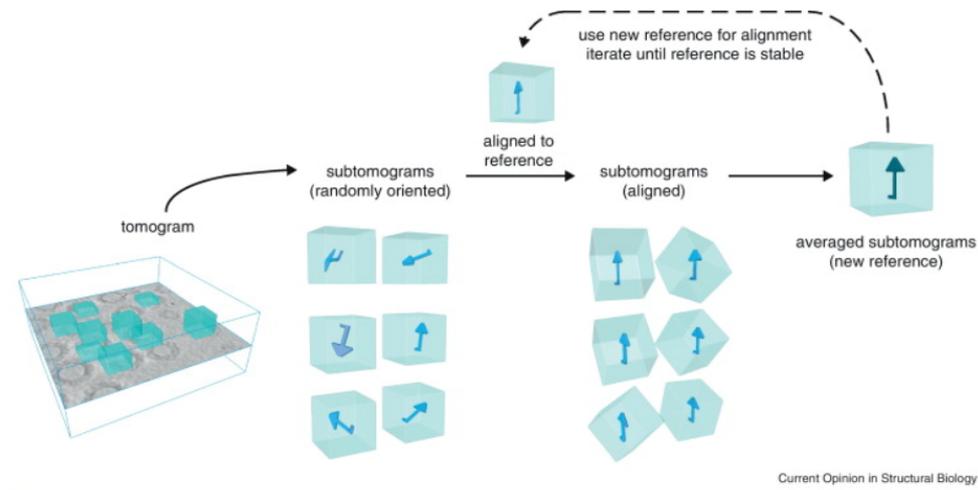
- Yes in random conical tilt and tomography



- Solution: combined different reconstructions with different missing cones

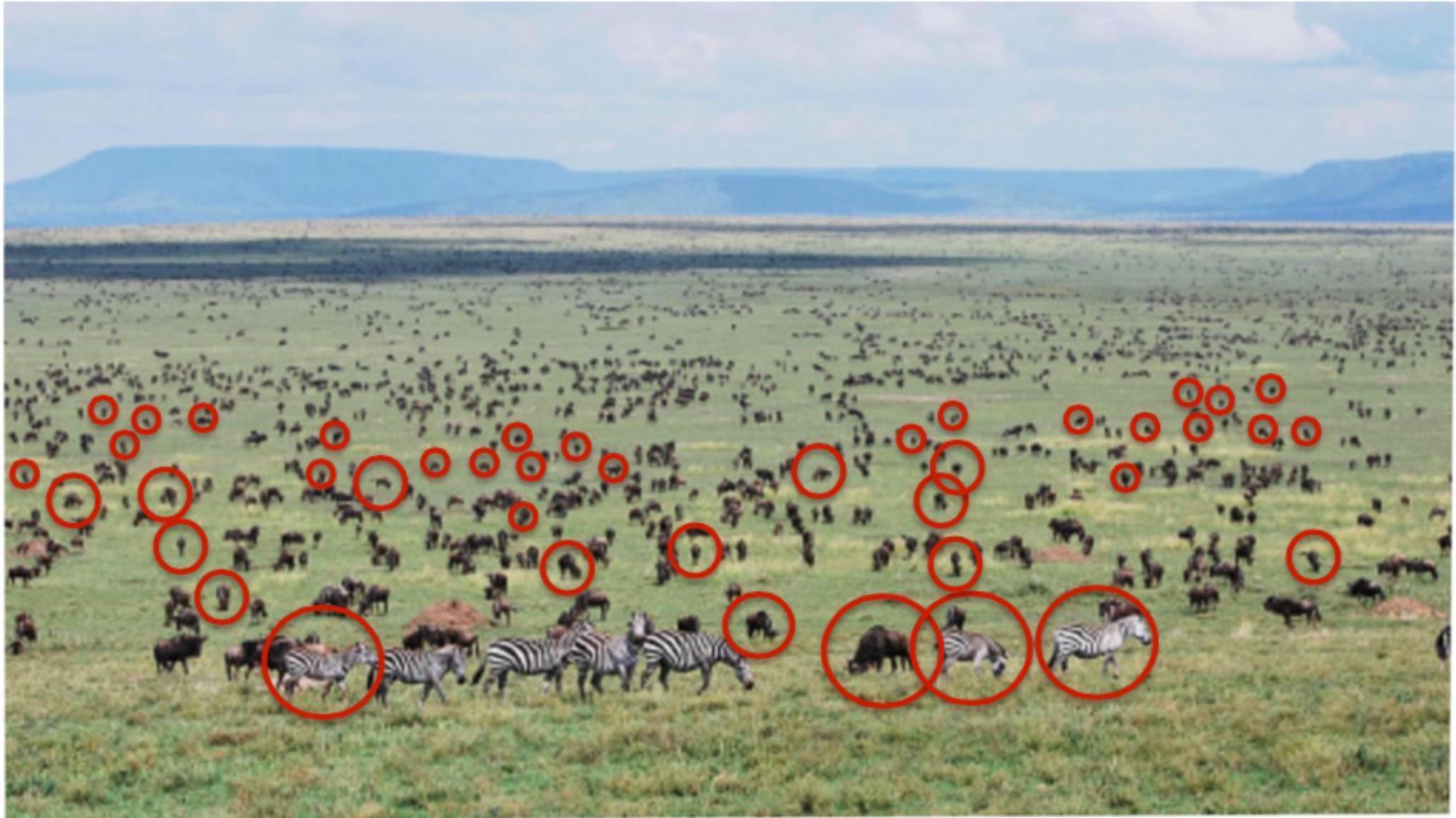


*From N. Boisset*



*From J. Briggs*

Caution: you can always get a 3D volume from your data



Caution: you can always get a 3D volume from your data

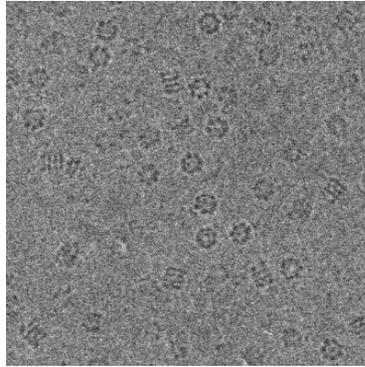


Great ! You got your 1<sup>st</sup> 3D reconstruction !!

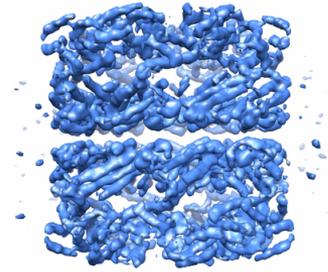
How do you make sure it is correct ?



**From 2D...**



**... To 3D**



- Step 1: determine orientations



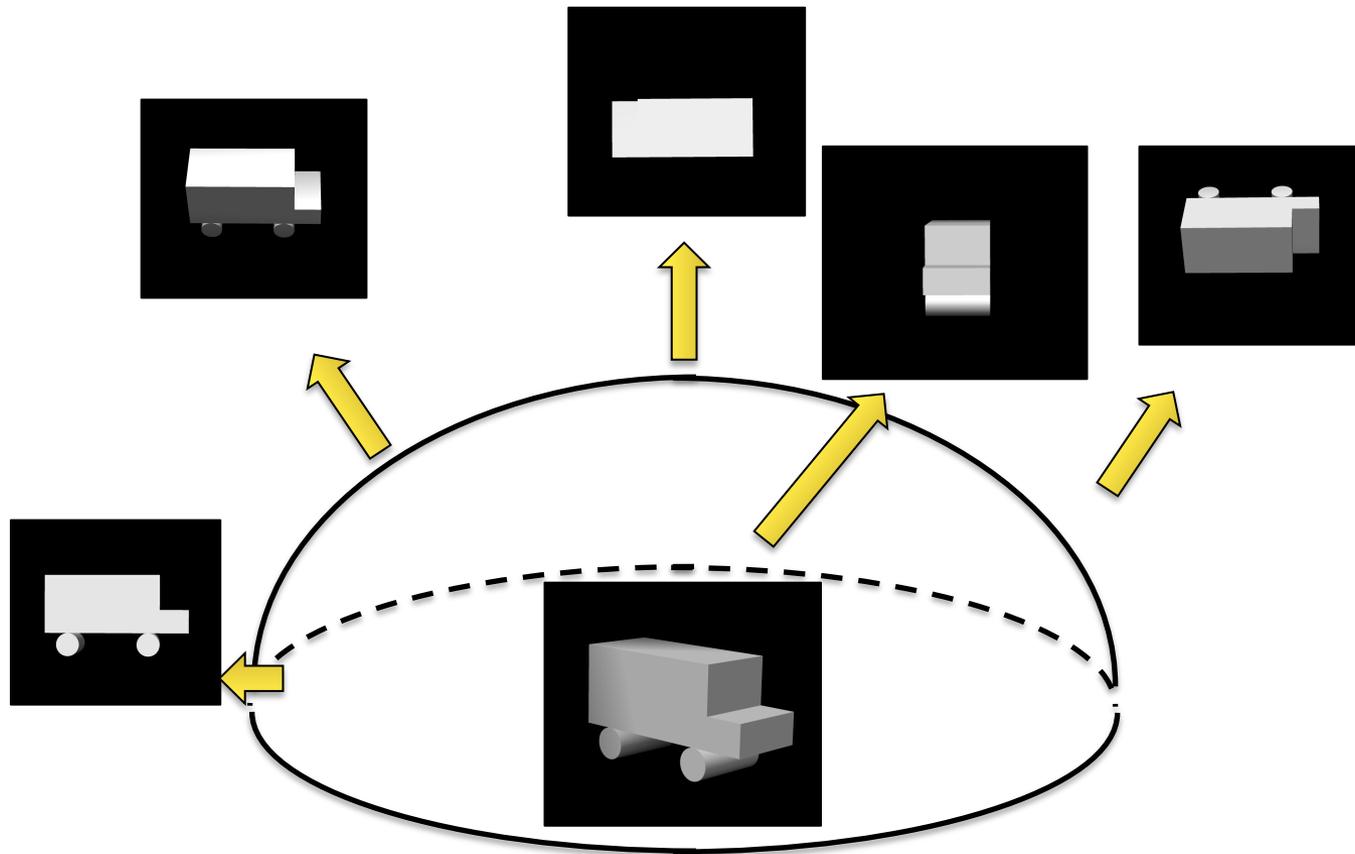
- Step 2: combine these orientations to obtain a 3D reconstruction



- Step 3: refine the structure

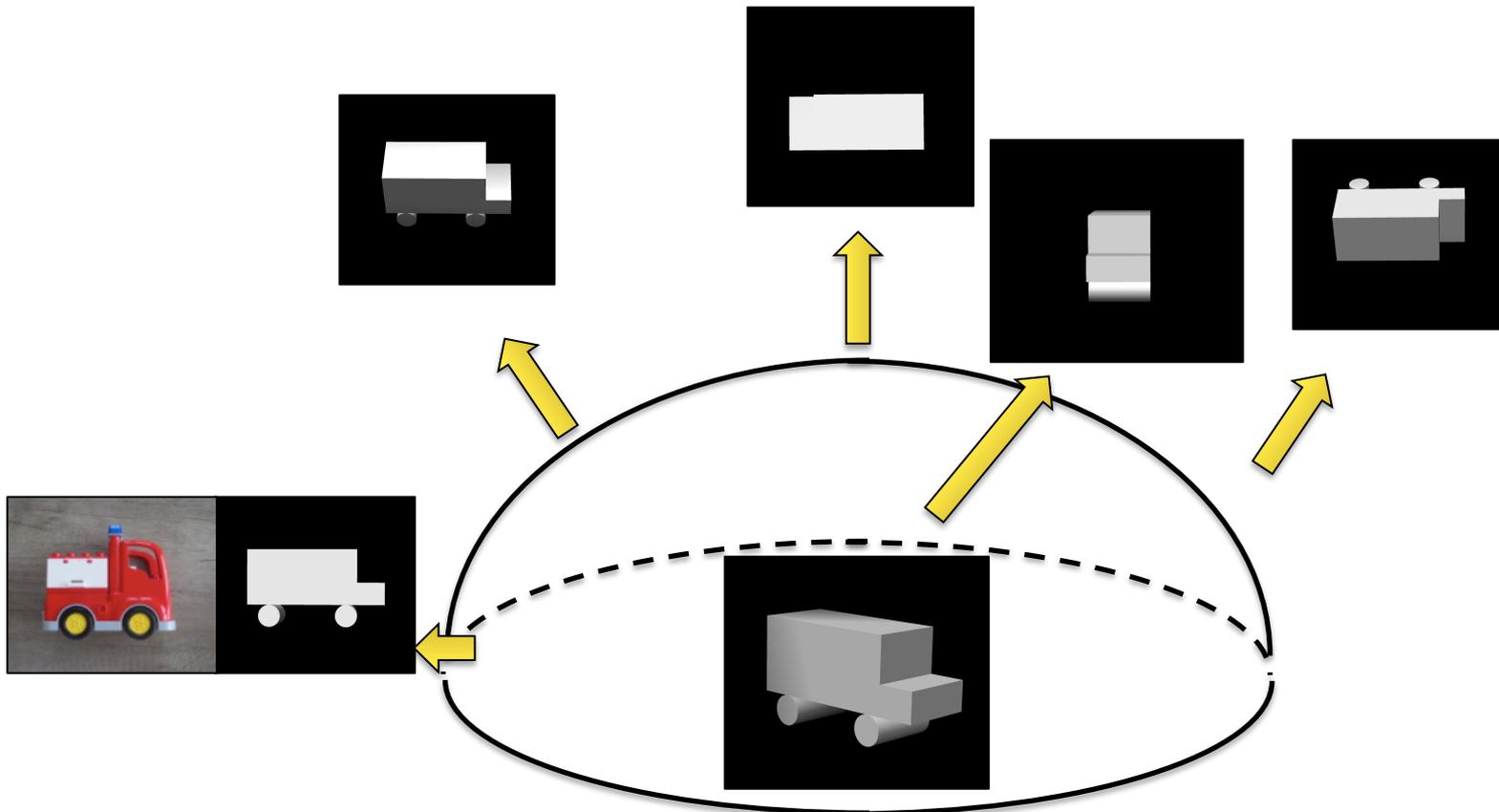
# Projections of the initial 3D reconstruction

- Project the initial 3D reconstruction towards all directions (of this asymmetric unit)
- Choose even distribution of projections.



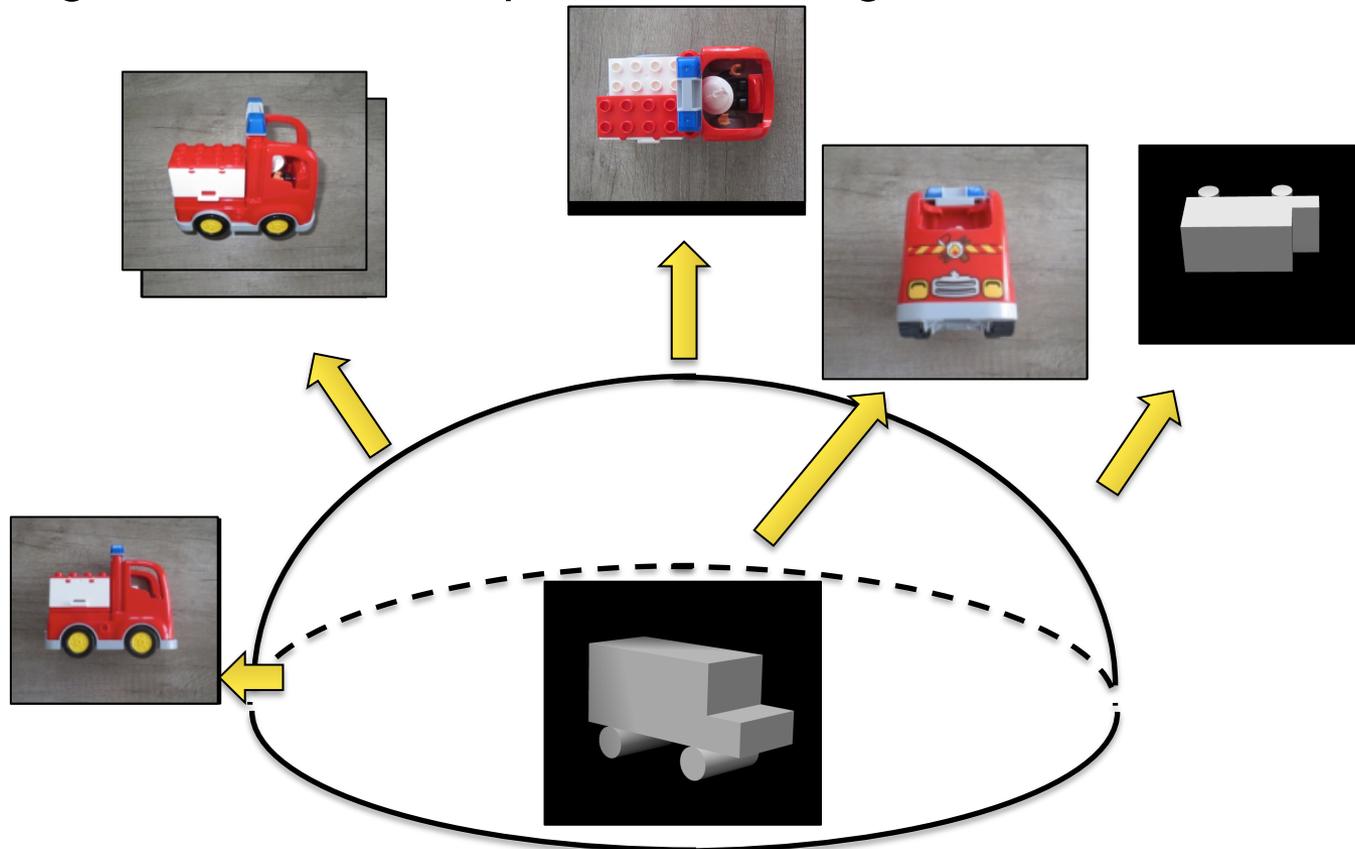
# Projection Matching, highest Cross-correlation criteria

- Compare each image to all the projections
- Allow translations and rotations of the image to find the best CC.
- Assign the euler angles of the projection having the highest CC to the experimental image.

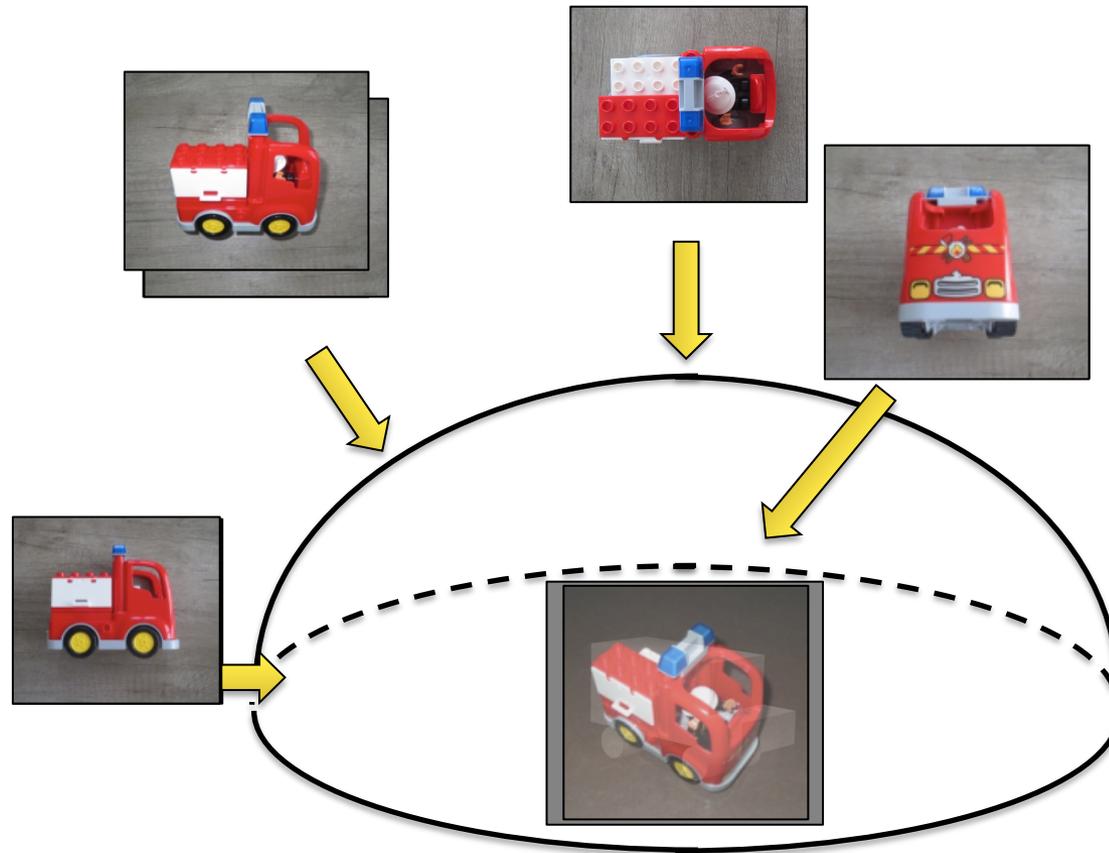


# Projection Matching, highest Cross-correlation criteria

- Compare each image to all the projections
- Allow translations and rotations of the image to find the best CC.
- Assign the euler angles of the projection having the highest CC to the experimental image.

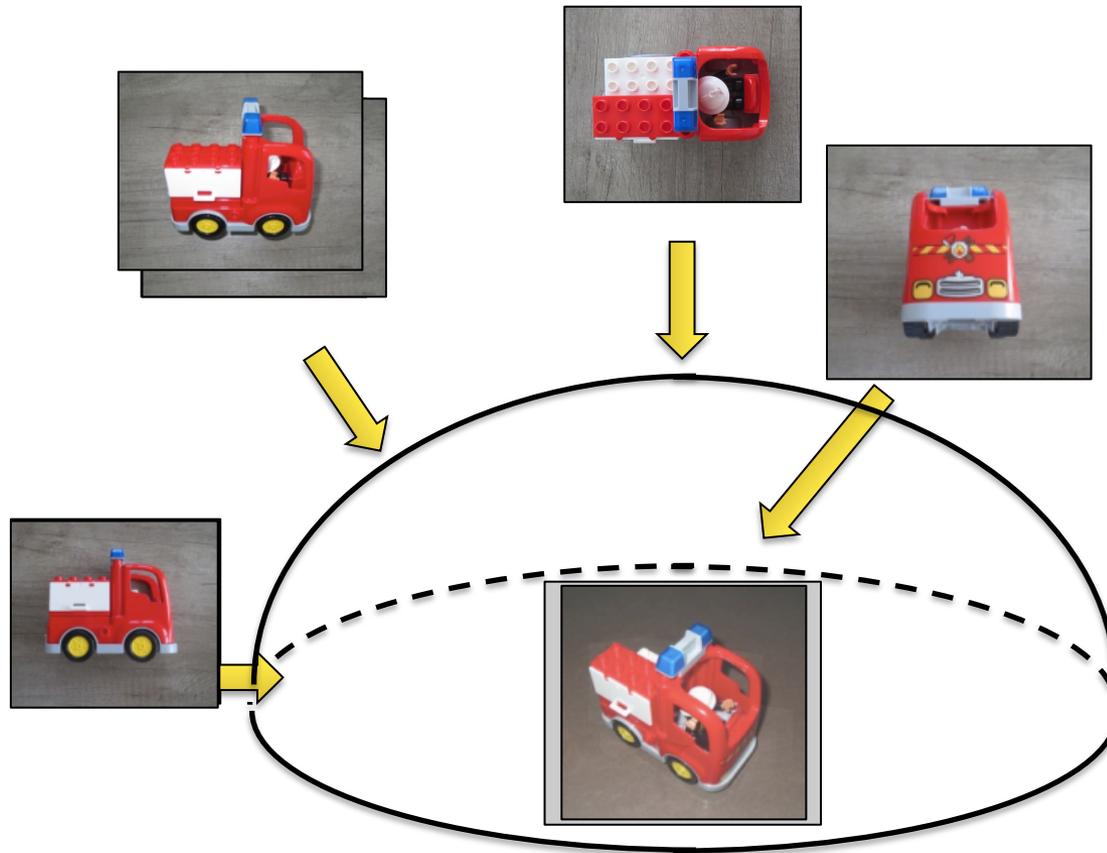


# Reconstruction by back projection

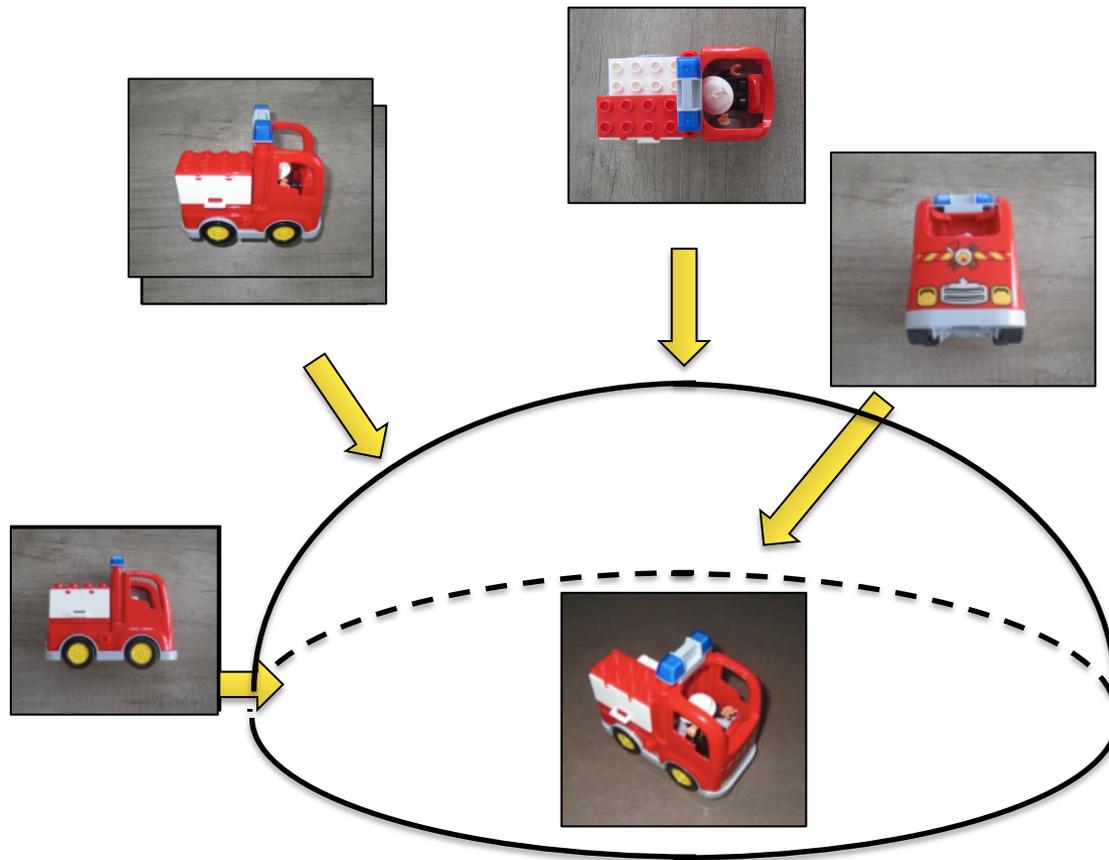


# Iterations

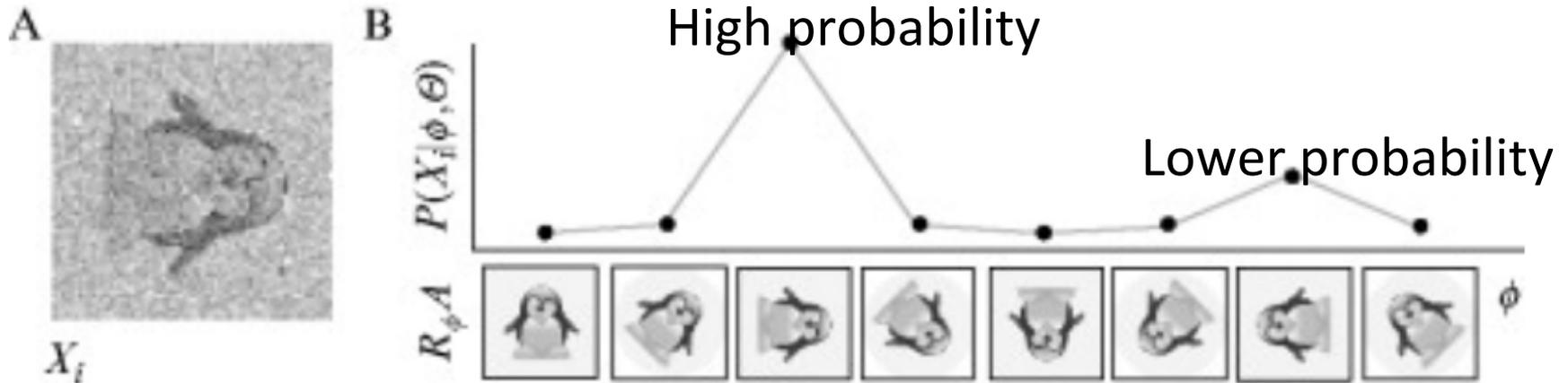
- Projections of the 3D reconstruction
- Projection matching with the highest cross-correlation criterion
- Back-projection



- The 3D reconstruction improves every iteration.
- Its projections are of better quality.
- Angular assignment is more and more precise until convergence.

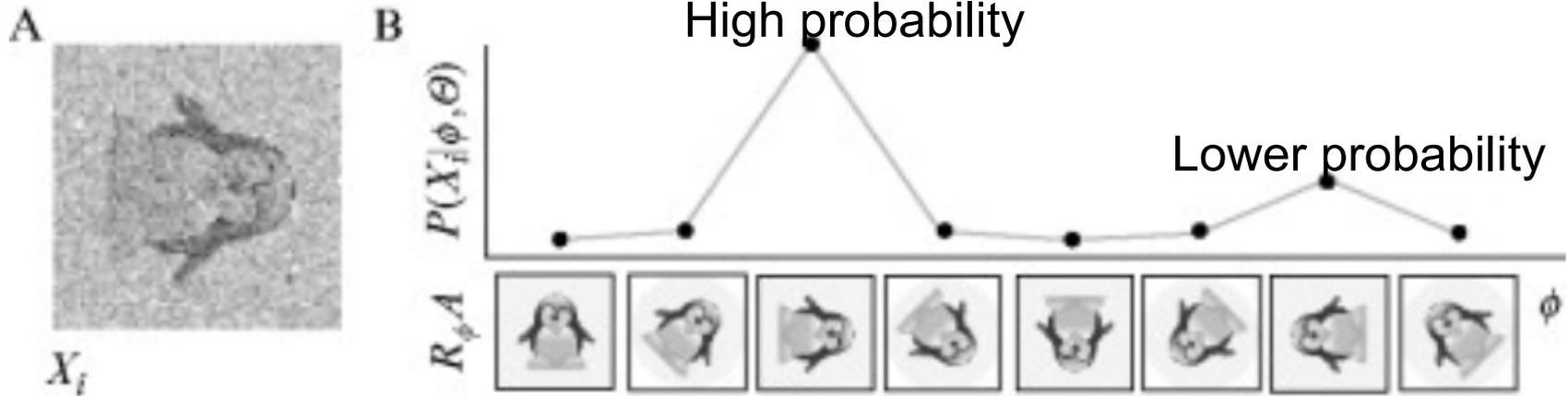


# Maximum likelihood methods applied to single-particle reconstruction

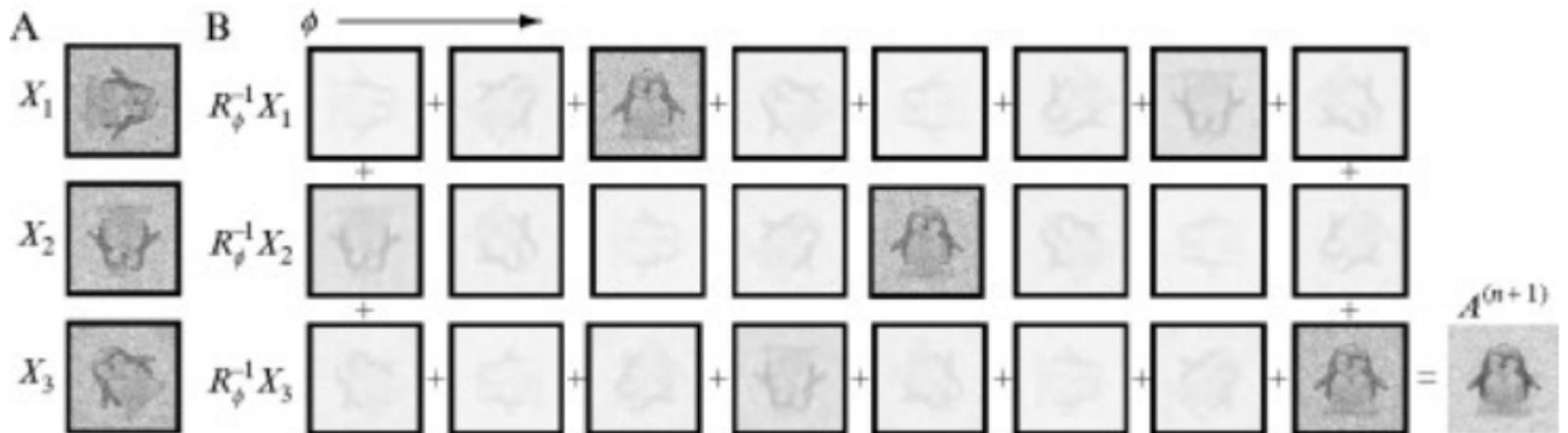


probability distribution function for orientations

# Maximum likelihood methods applied to single-particle reconstruction



probability distribution function for orientations



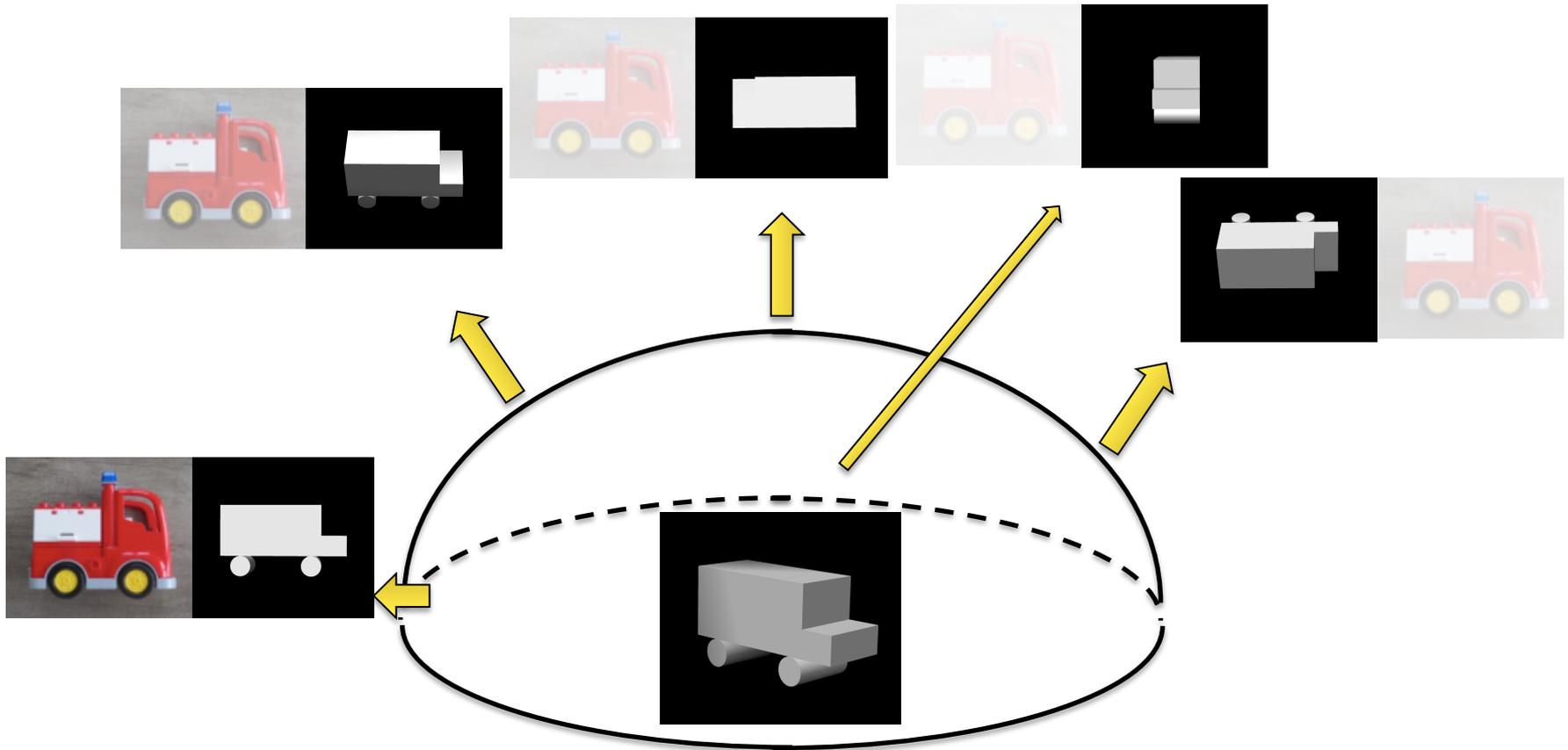
probability-weighted averaging

*From Scheres*

➔ Cleaner references, higher radius of convergence

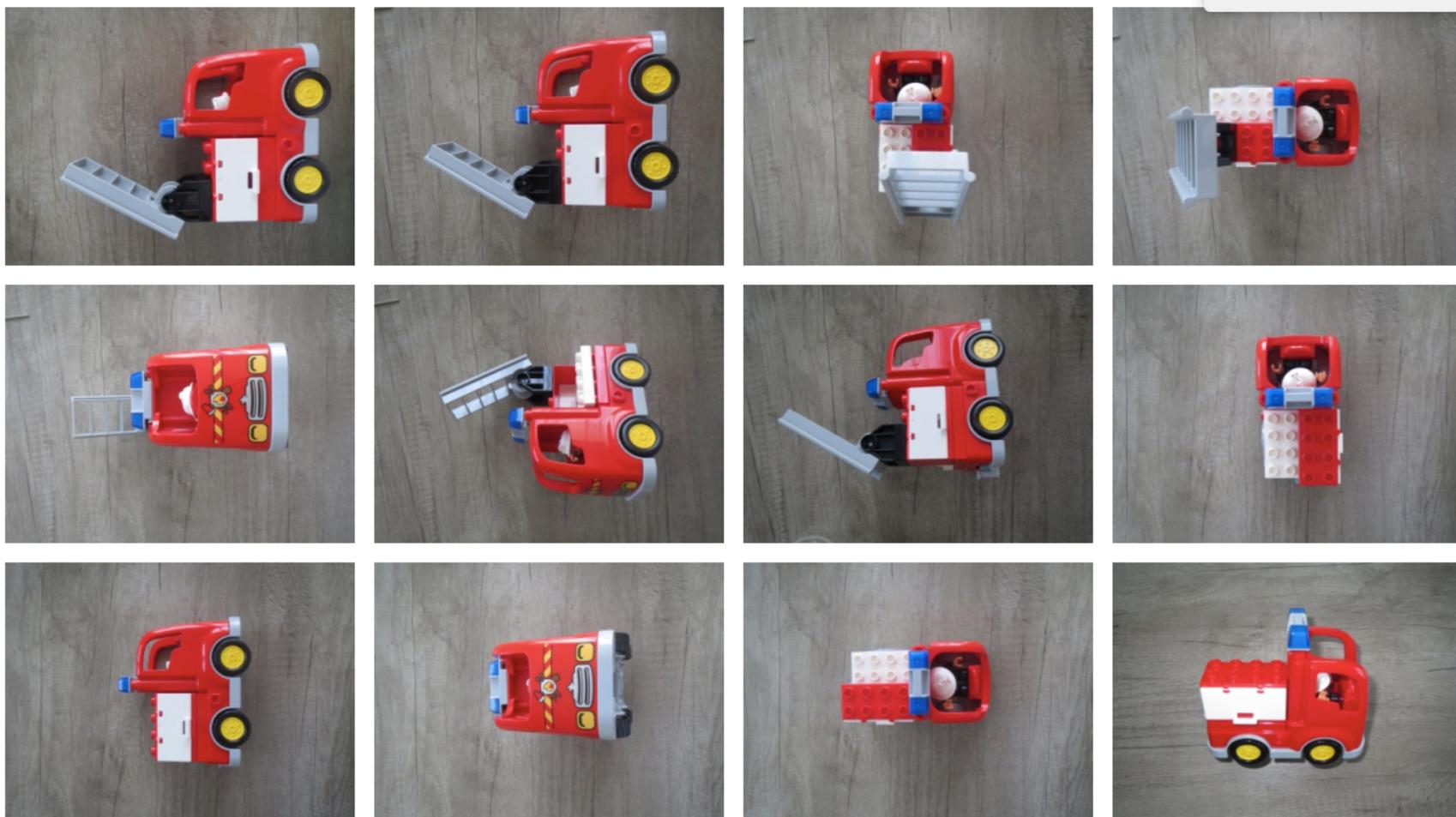
# Maximum-likelihood applied to our example

- For each image, a probability is given to every orientation
- More robust as we have images with very low signal to noise

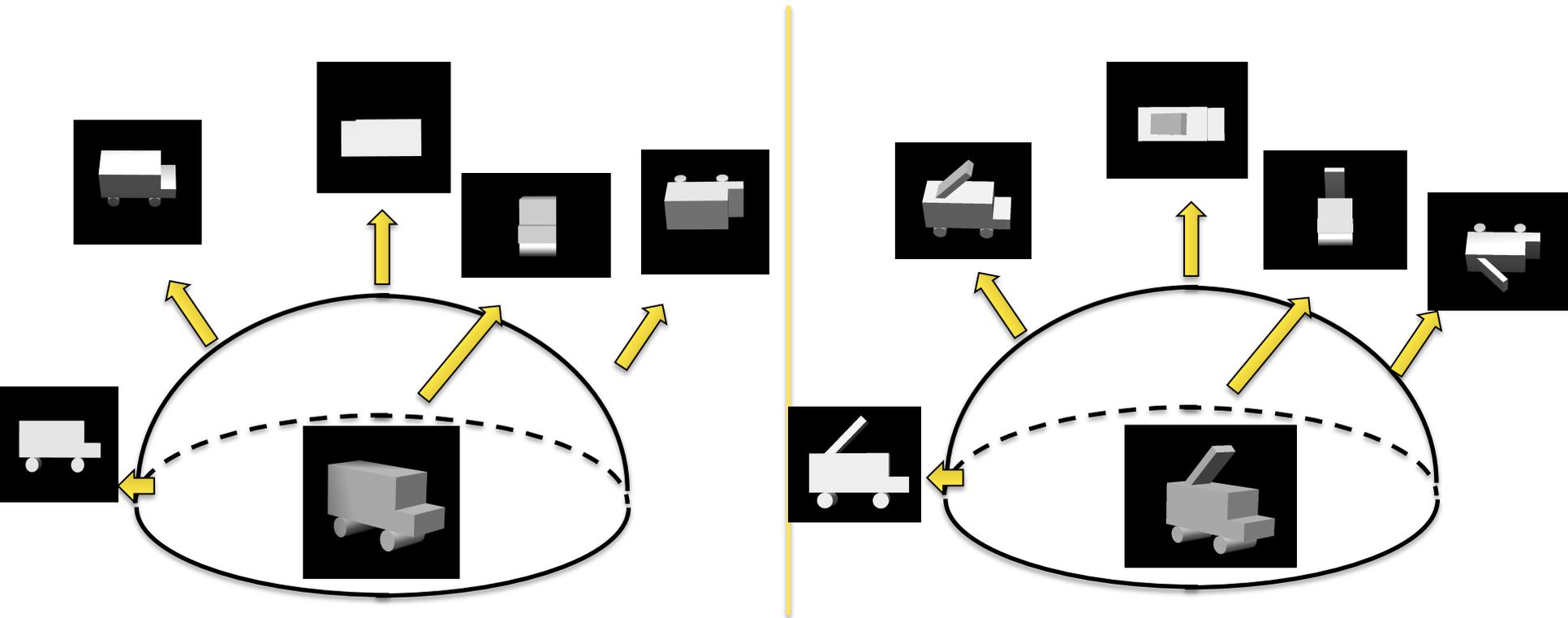


## How to deal with structural heterogeneity ?

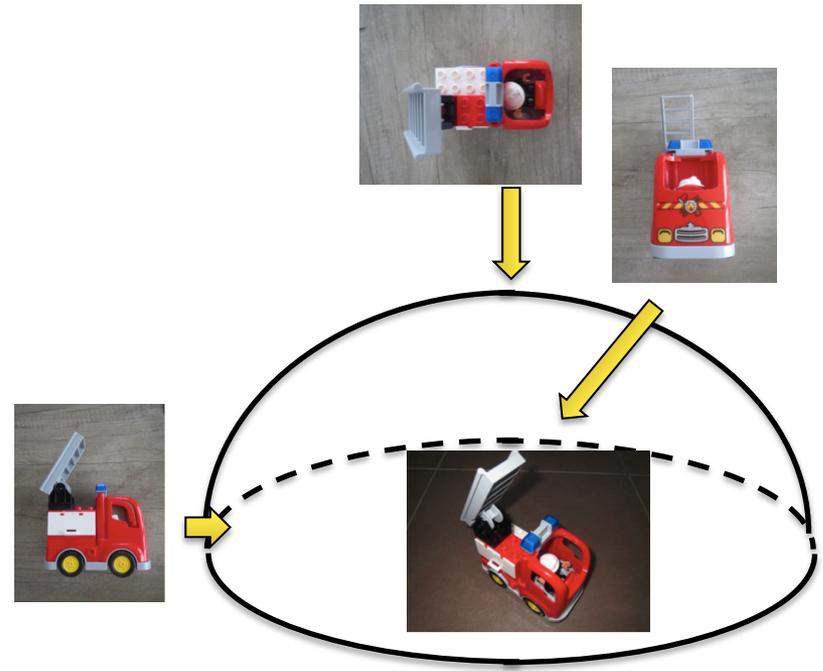
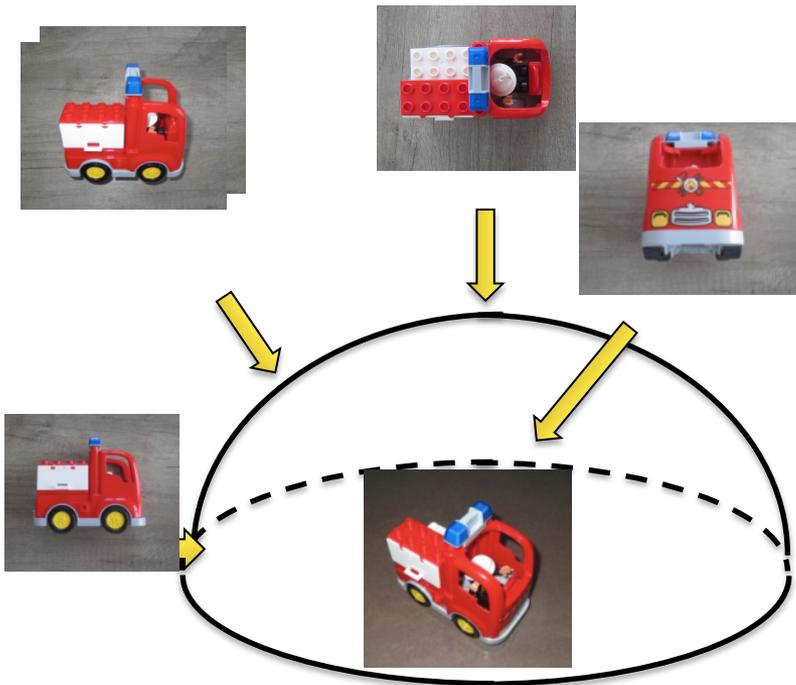
- Most datasets are, to some extent, heterogeneous.
- For example, you can have a dataset with your complex in both active and inactive states.



- Cryo-EM image analysis allows to classify these 3D heterogeneities
- Generation of X initial reconstructions (can be obtained from random subsets of images with assigned euler angles)
- « Competitive » projection matching.
- Can be done with the highest cross-correlation criterion or with maximum likelihood.

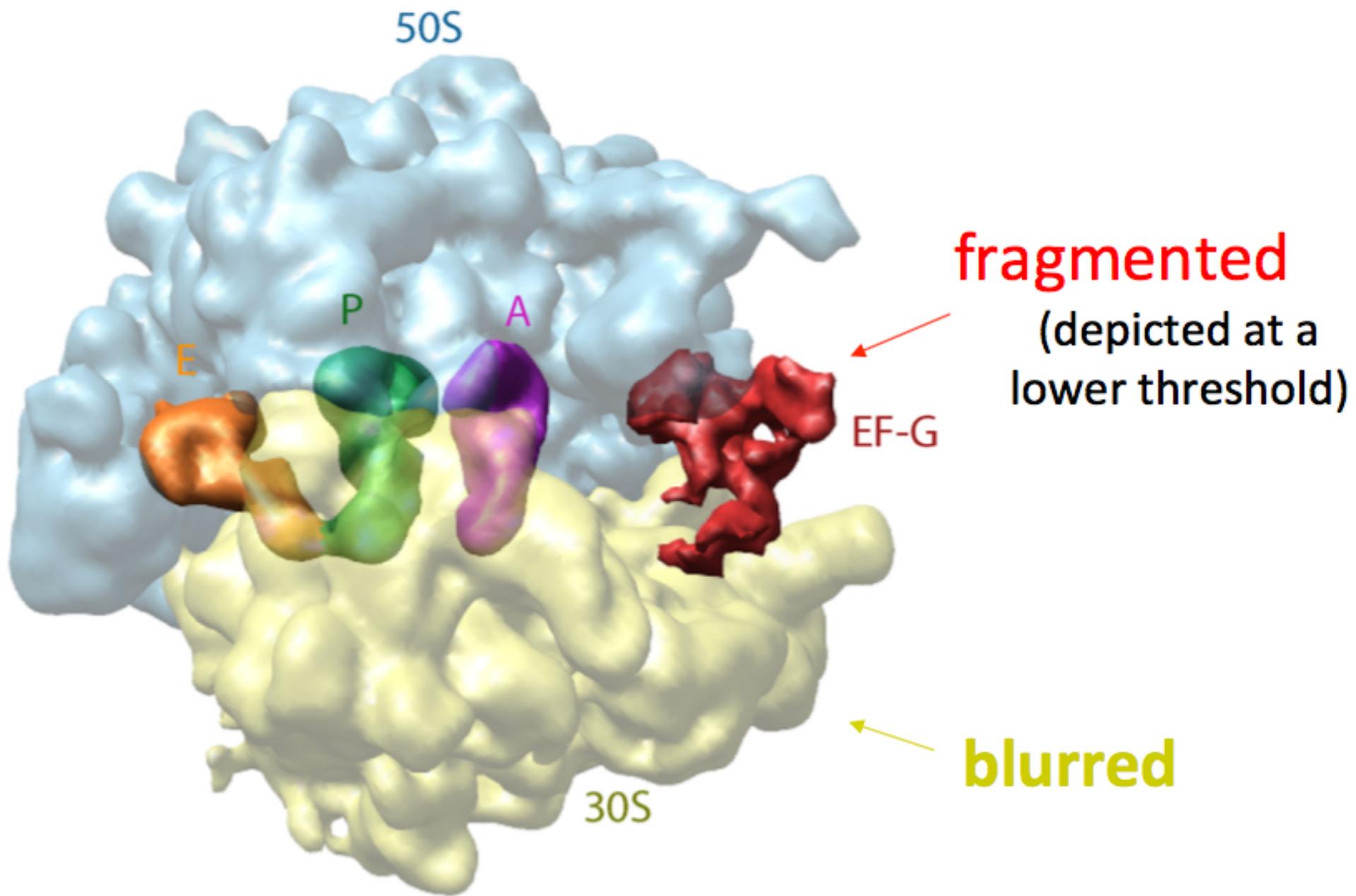


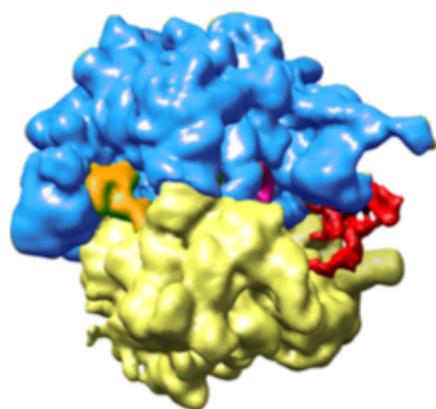
- Allows to obtain several structures out of one dataset !
- As subsets are more homogeneous, they can reach higher resolution !



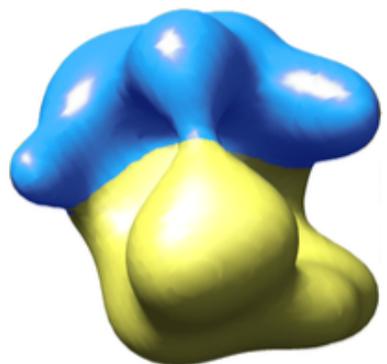
## Methods to classify 3D heterogeneity

- « Competitive projection matching » using the highest cross-correlation criteria
- « Competitive projection matching » using maximum likelihood
- 3D MSA

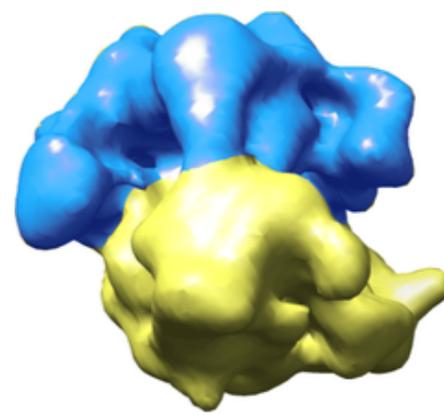
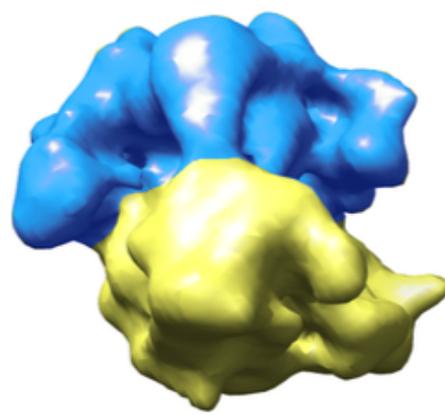
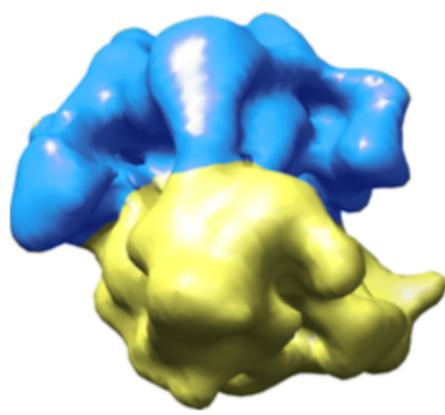
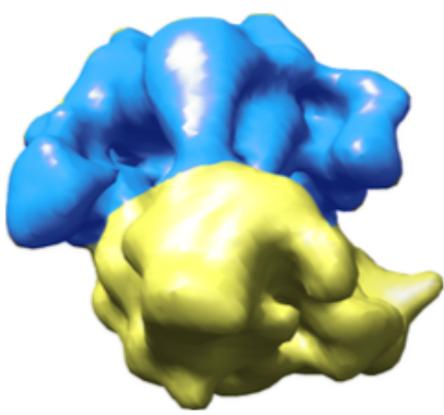


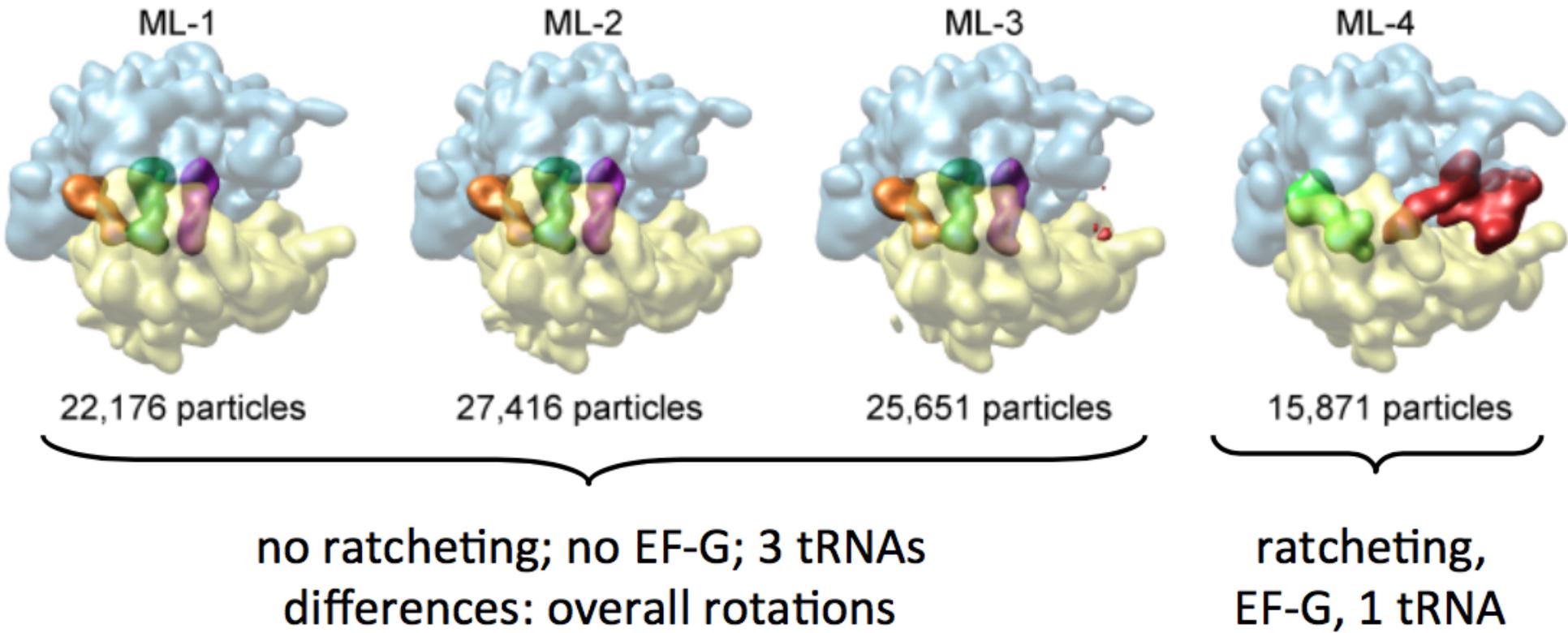


80 Å  
filter



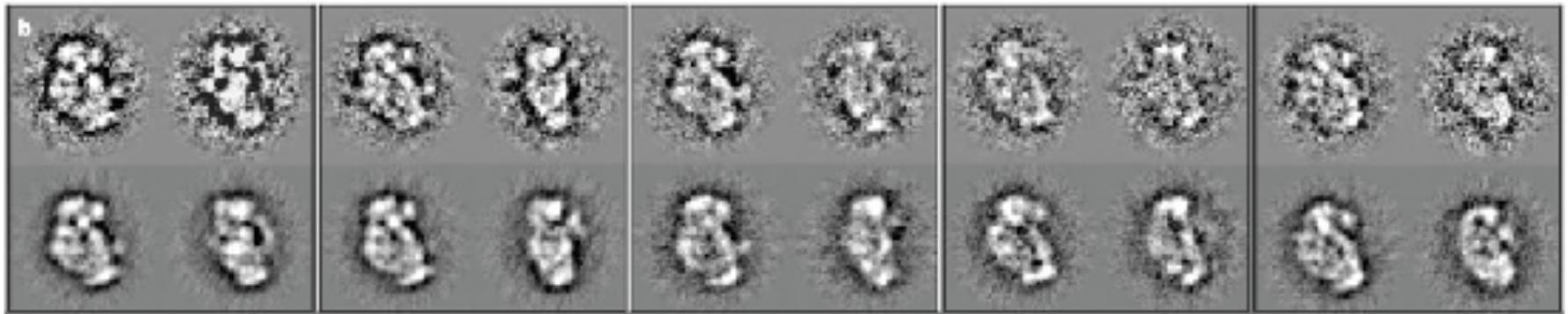
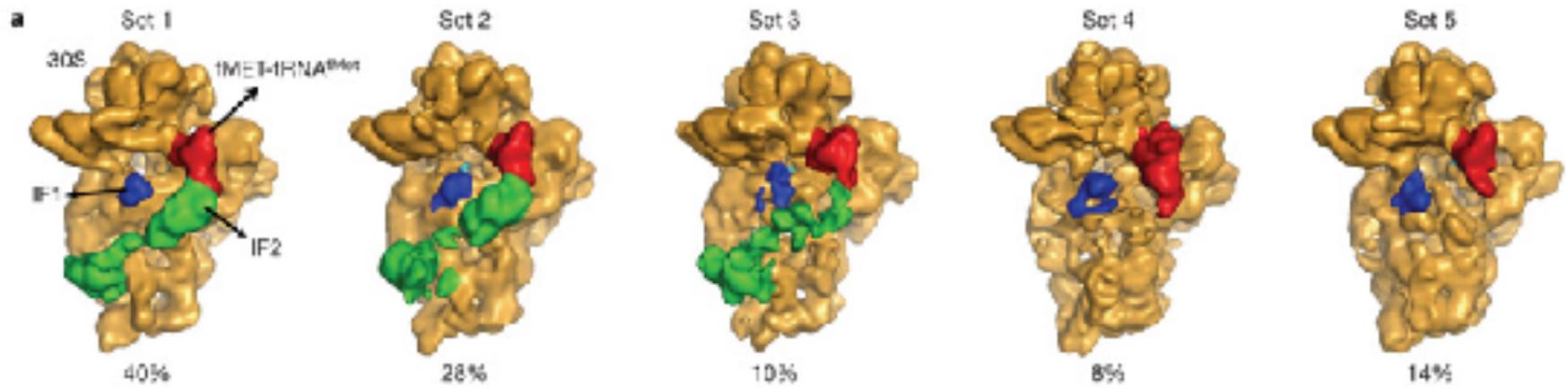
4 **random** subsets; 1 iter ML



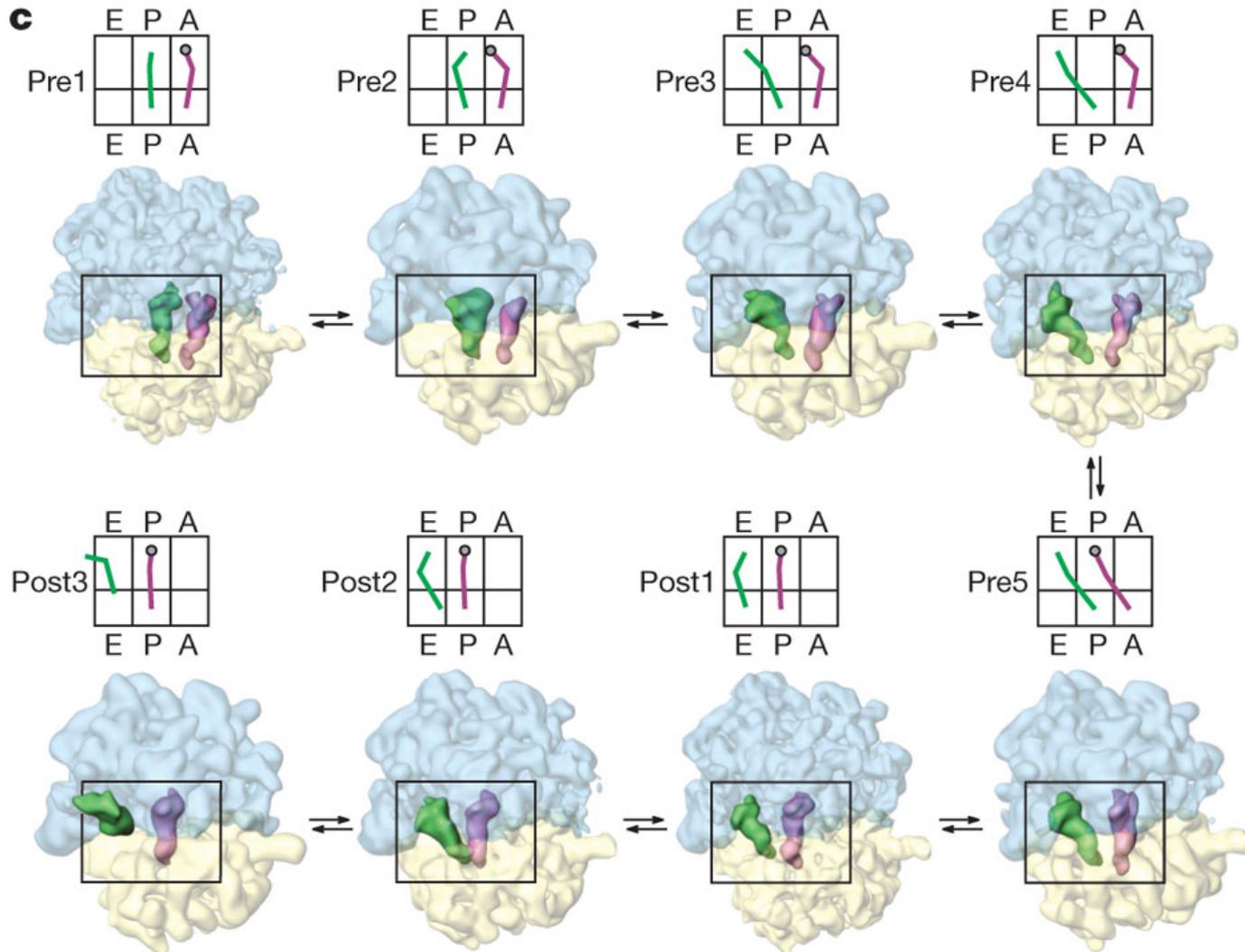


(Results coincided with a supervised classification)

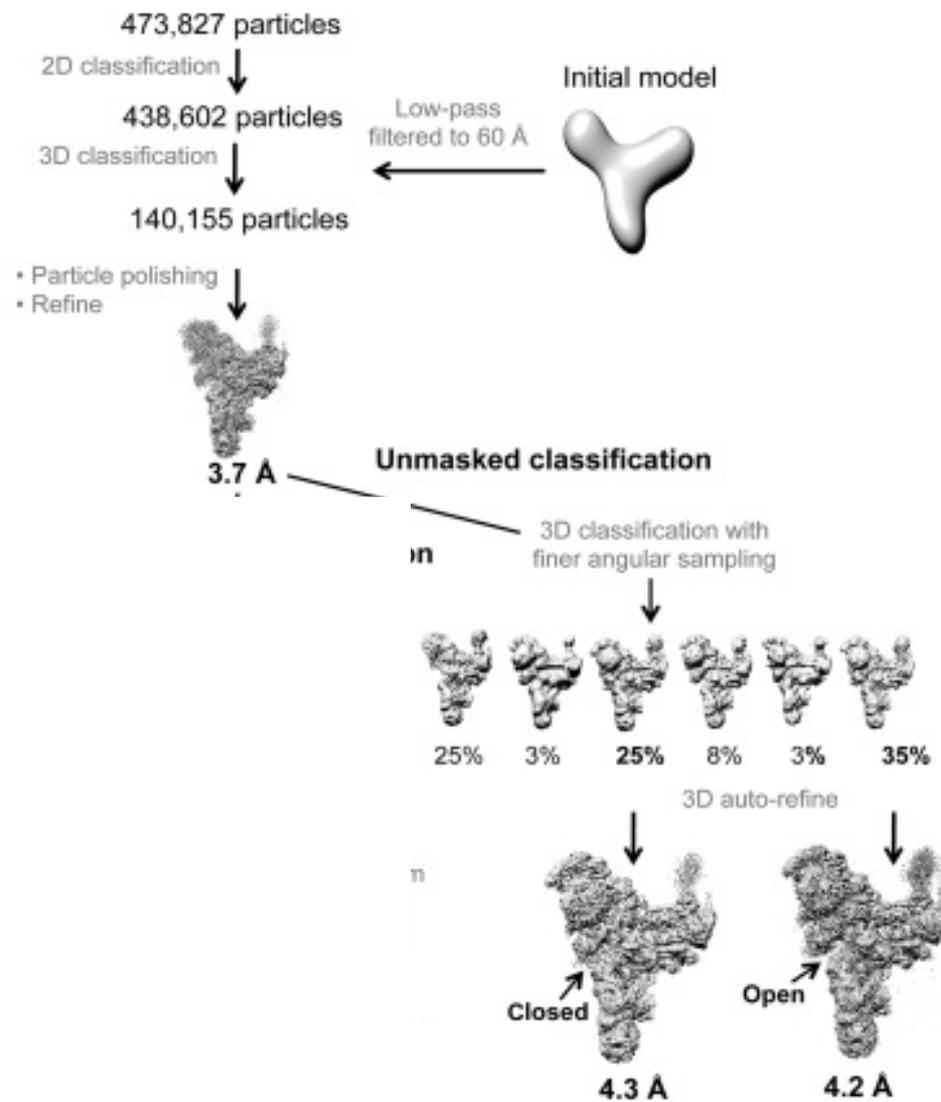
Scheres et al (2007) Nat. Meth.



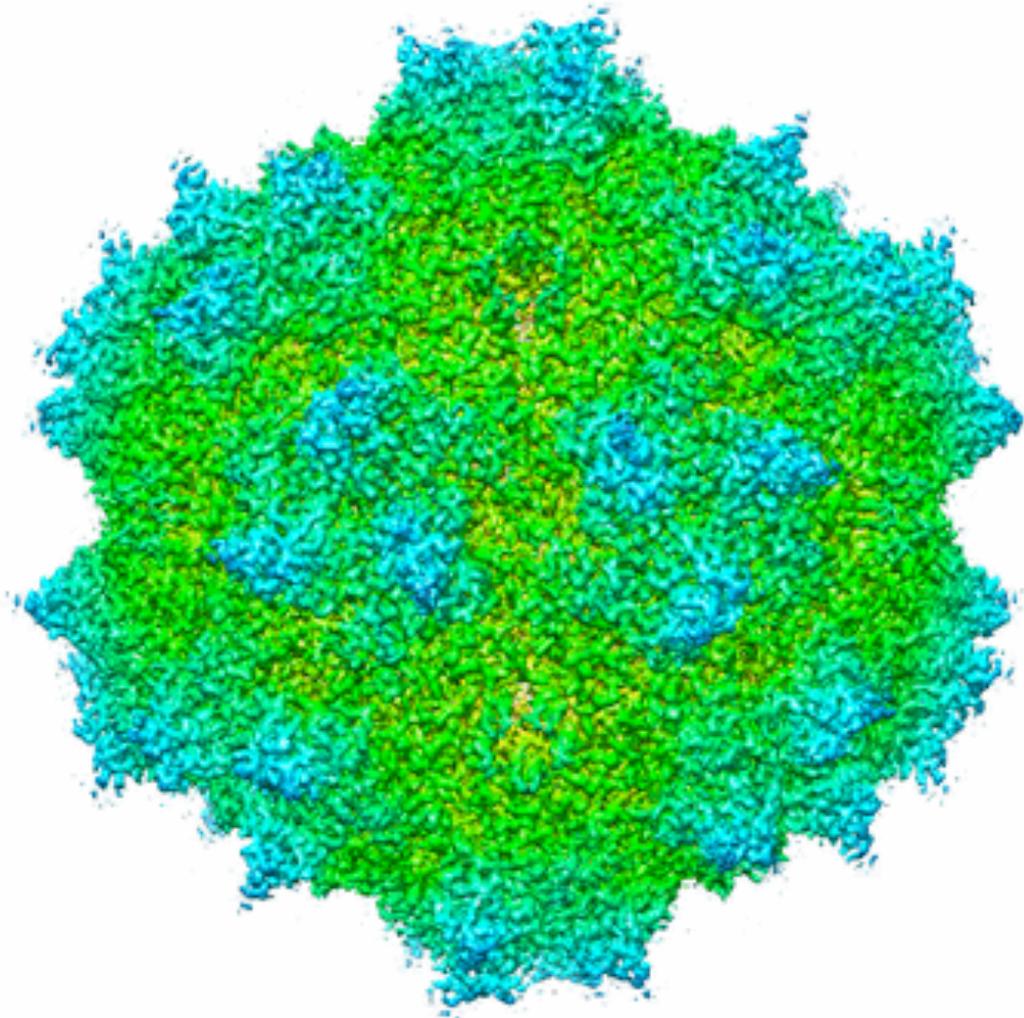
# Separation of time-resolved states



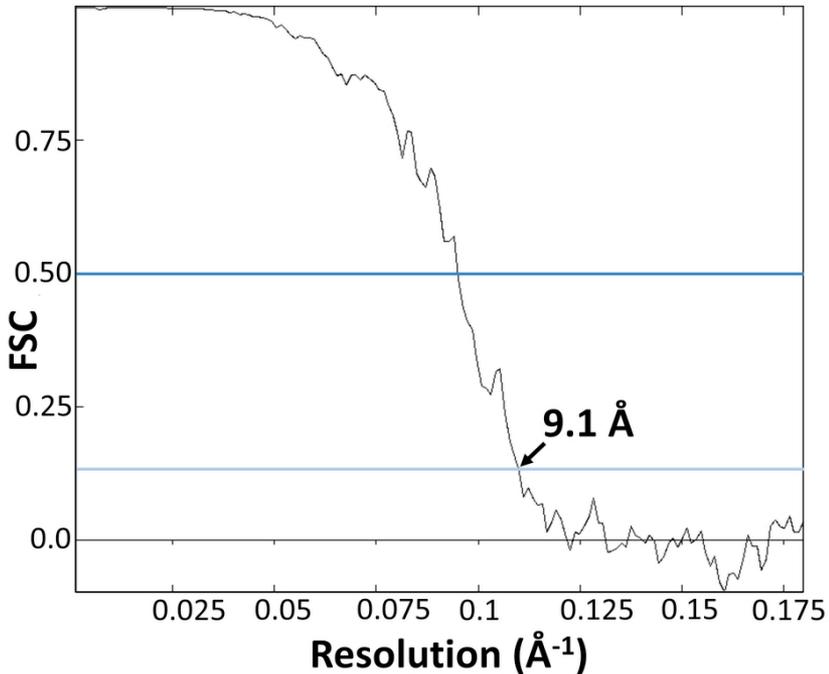
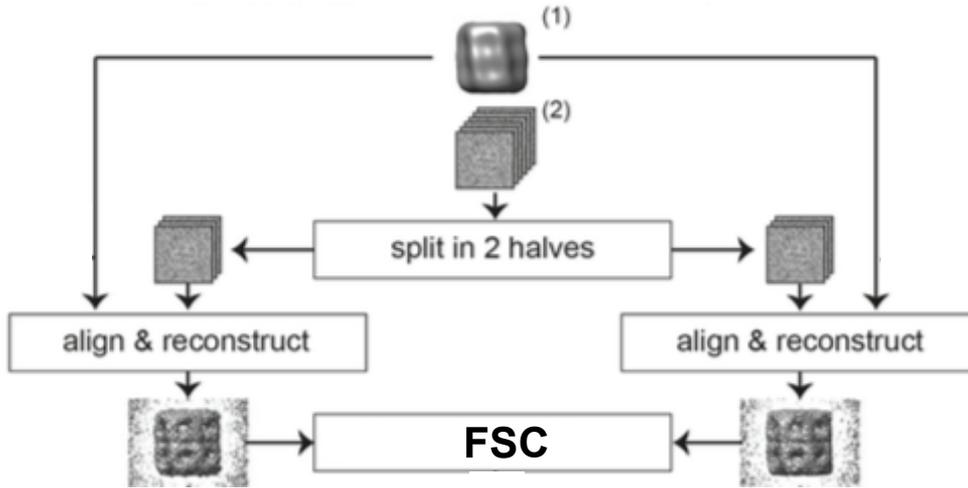
Look how great EM image analysis can become !!



**Congratulations ! You got your 1<sup>st</sup> refined EM map !**



# How to calculate the resolution?



Dataset separation in two halves  
↓  
One reconstruction from each half  
↓  
Comparison in Fourier space for each Fourier shell

Several criteria:

0.5

0.143

3 sigma

$\frac{1}{2}$  bit

*Harauz and van Heel (1986)*  
*Rosenthal et al., 2003*

# Resolution is not the same everywhere, local resolution can be calculated

