

Basis of the Small Angles X-rays Scattering method

Pierre Roblin

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1997-2001 : Maîtrise de biologie structurale Université Paul Sabatier

- *Résolution de structures par cristallographie des rayons X : β -Lactamases (Laurent Maveyraud, IPBS équipe biocristallographie J.P. Samama)*

2001-2003 : Institut National des Sciences Appliquées de Toulouse. Bio-ingénierie des protéines option biologie structurale

- *RMN 2D sur des peptides de synthèse (James H. Davis, Université de Guelph, Ontario) et (Virginie Gervais, IPBS équipe RMN structurale A. Milon)*

2002-2003 : DEA de biologie structurale Université Paul Sabatier

- *Résolution de structures par cristallographie des rayons X : leucotoxine *S. aureus* (IPBS équipe biophysique structurale L. Mourey)*

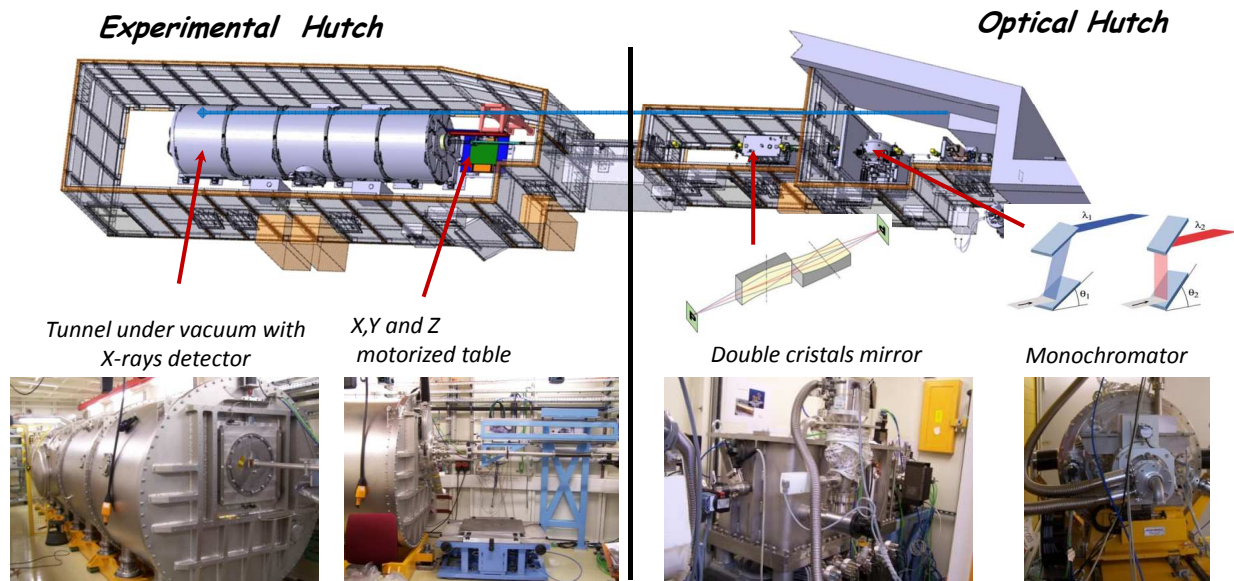
2003-2007 : Thèse de biologie structurale Université Paul Sabatier

- *RX + SAXS : polykétide synthase Myco. Tub (IPBS équipe biophysique structurale L. Mourey) et (D. Svergun EMBL Hambourg)*

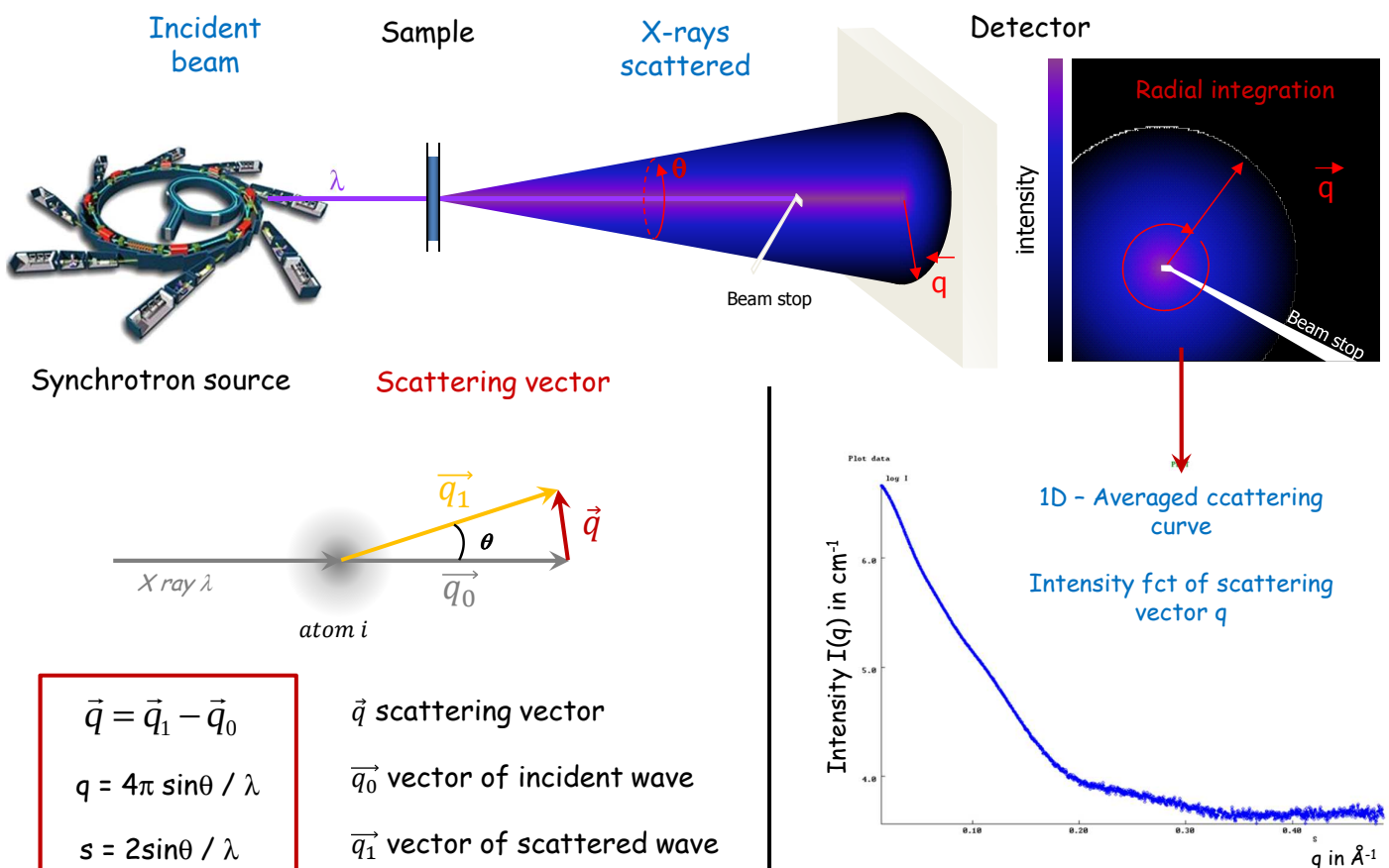
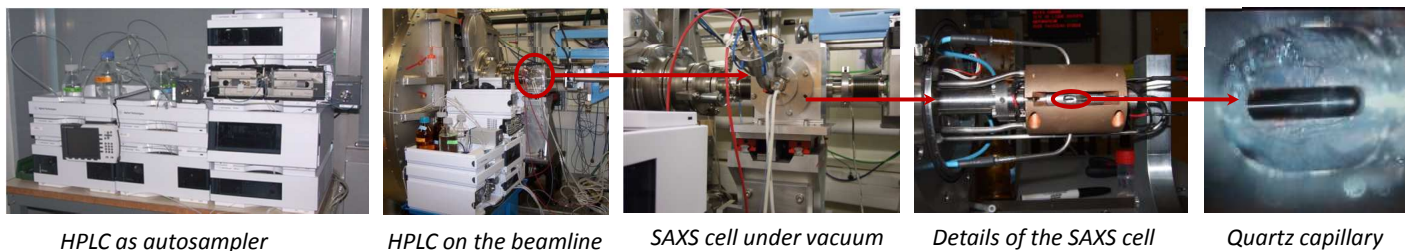
2007-2009 : Post doc L.E.B.S. Gif sur Yvette

- *RX + SAXS : complexe actine (Louis Renault équipe MF Carlier)*

Depuis 2009 : Ingénieur de recherche INRA, mise en disponibilité au synchrotron SOLEIL, ligne SWING



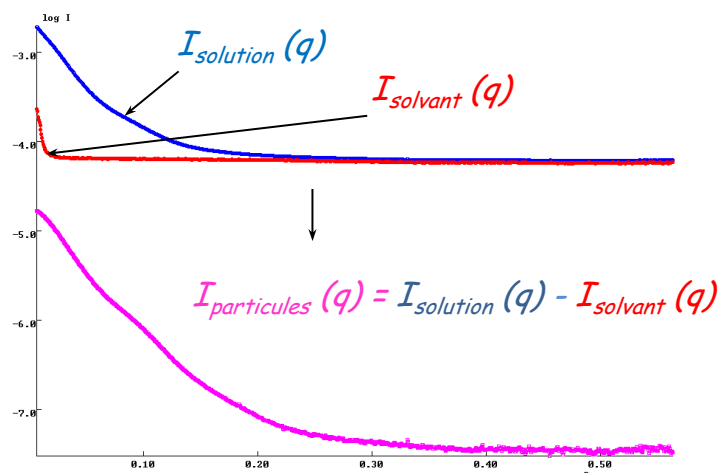
Sample environment dedicated to the biology :



Method sensible to the difference between the electronic density of the particle and the solvent (contrast)



$$I_{\text{solution}}(q) = I_{\text{solvent}}(q) + I_{\text{particles}}(q)$$



Structural information obtained from scattering curve

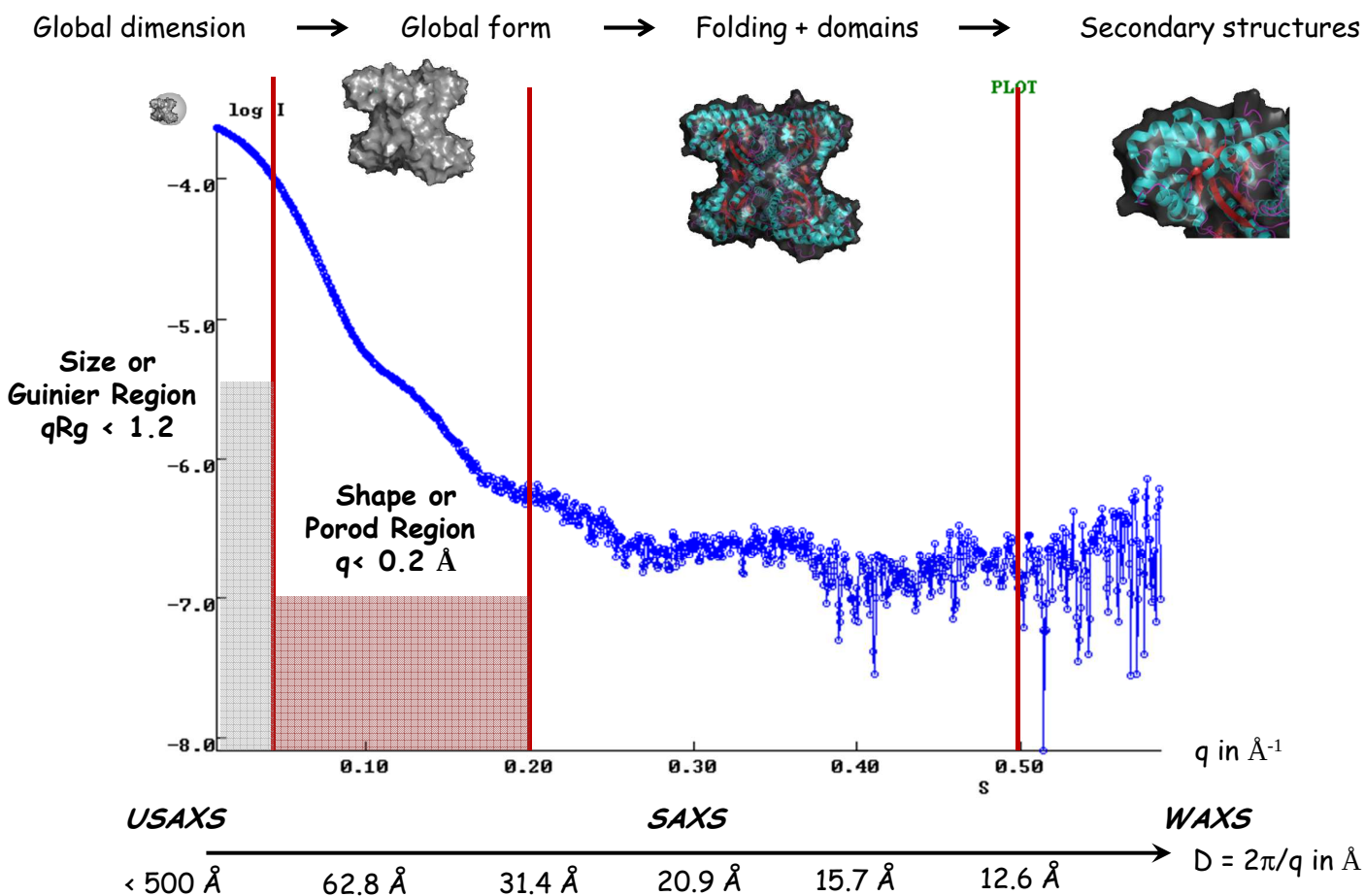
- biophysical parameters (size and shape of the object)
- molecular weight, oligomerization state and volume
- low resolution molecular shape calculation with *ab initio* method
- comparison with high resolution model
- molecular modeling of unstructured missing part
- molecular modeling rigid body of complex

Biophysical informations calculated directly from the SAXS curve

3D structural informations

SAXS data compatible model
NOT a structure

A typical SAXS solution curve of protein



Close to $q=0$, the scattering intensity of a particle can be described by a Gaussian curve.

The Guinier law is equivalent of a linear variation of $\ln(I(q))$ vs q^2 (Guinier plot), providing R_g and $I(0)$. The validity domain actually depends on the shape of the particle and is around $q < 1.3/R_g$ for a globular shape.

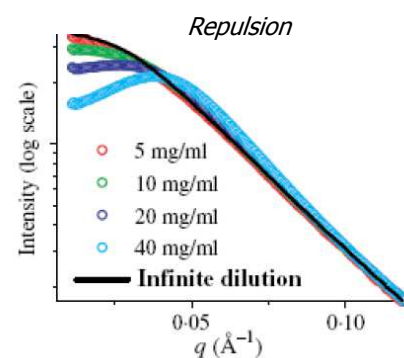
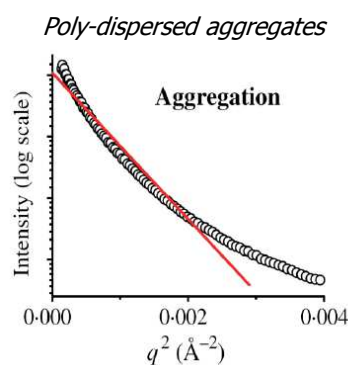
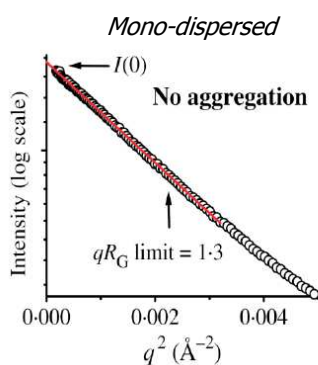
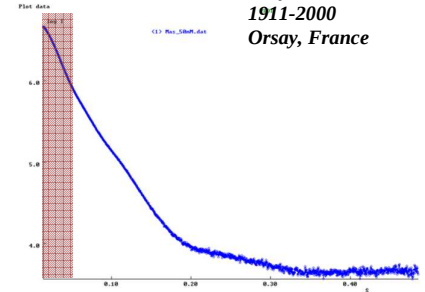


Prof. André Guinier
1911-2000
Orsay, France

$$I(q) = I(0) \exp\left(-\frac{q^2 R_g^2}{3}\right) \quad \text{or} \quad \ln[I(q)] = \ln[I(0)] - \frac{q^2 R_g^2}{3}$$

Extrapolated intensity at origin

Radius of gyration



Putnam, D., et al. (2007) *Quart. Rev. Biophys.* 40, 191-285.

Determination of the mass from Guinier law

From extrapolated intensity at the origin $I(0)$, the molecular mass can be determined with the following equation :

$$I(0) = \frac{c \cdot M}{N_A} \cdot r_0^2 \cdot [v_p (\rho_{prot} - \rho_{buf})]^2$$

Mass concentration

Classical electron radius

Protein specific volume

Electronic density contrast

$$M_{\text{exp}}(\text{kDa}) = I(0) \cdot N_A / [c \Delta \rho_m^2]$$

with $\Delta \rho_m \approx 2 \cdot 10^{10} \text{ cm}^3/\text{g}$ for proteins
and $I(0)$ in cm^{-1}

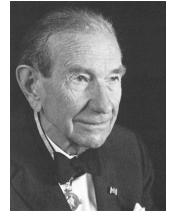
$I(0)$ gives an independent estimation of the molar mass of the protein
(only if the mass concentration, c , is precisely known ...)

$I(0)$ must be expressed in absolute units cm^{-1} necessitating a calibration before.

$$\text{Typically : } M (\text{kDa}) = 1500 \cdot I(0) (\text{cm}^{-1}) / C (\text{mg/ml})$$

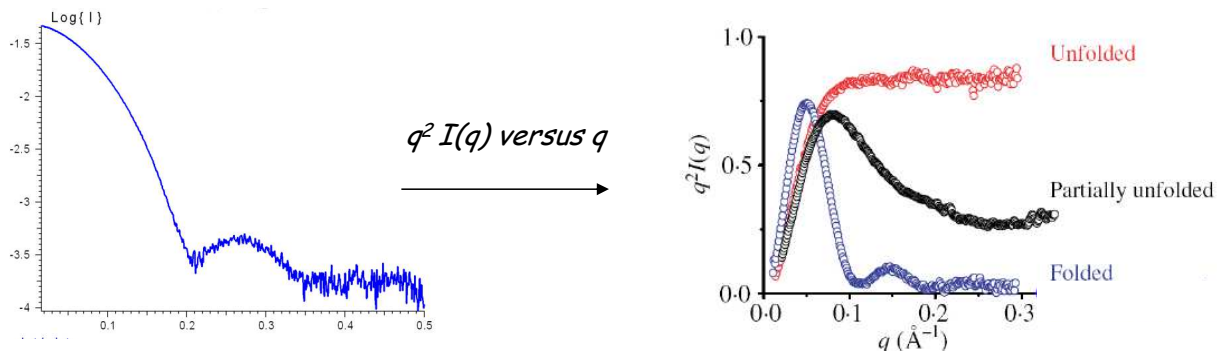
SAXS provides a sensitive means to *evaluate the degree of compactness* of a protein:

- o To determine whether a protein is globular, extended or unfolded
- o To monitor the folding or unfolding transition of a protein



Prof. Otto Kratky
1902-1995
Graz, Austria

This is most conveniently represented using the so-called Kratky plot:



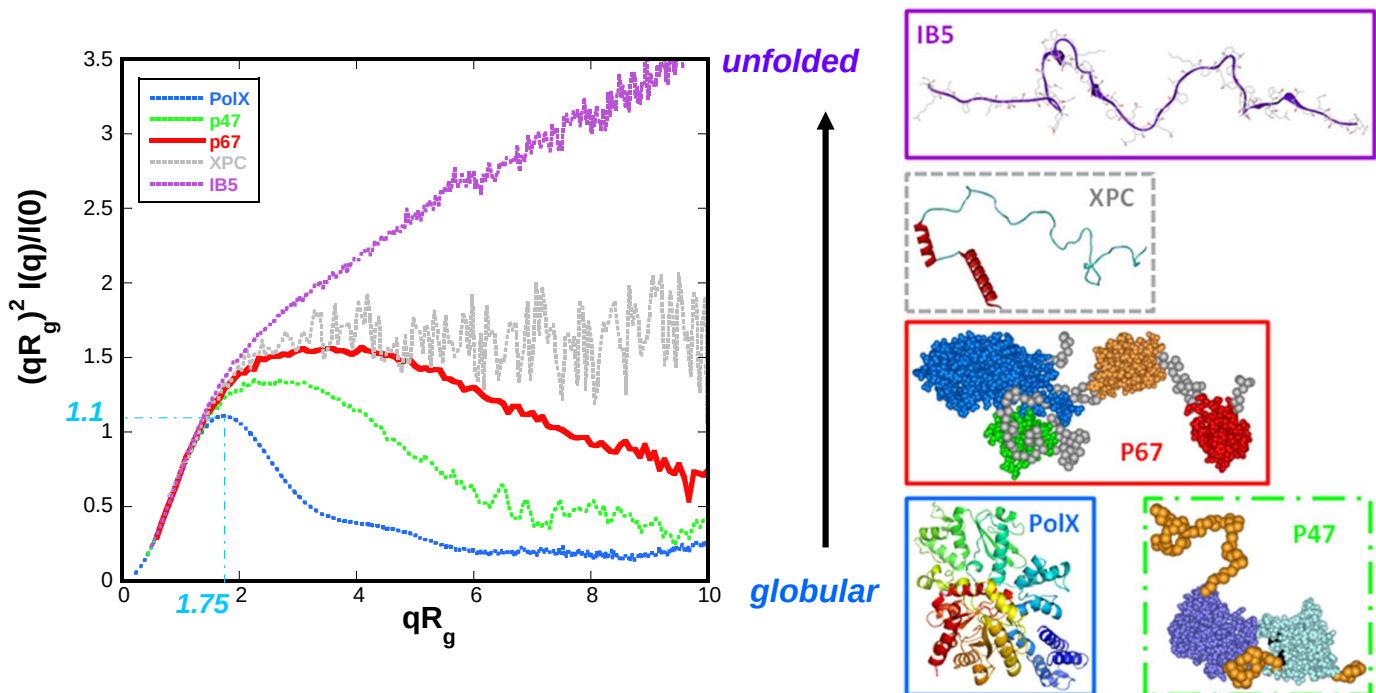
Putnam, D., et al. (2007) *Quart. Rev. Biophys.* 40, 191-285.

Folded particle : *bell-shaped curve* (asymptotic behaviour in $I(q) \sim q^{-4}$)

Random polymer chain : *plateau* at large q -values (asymptotic behaviour in $I(q) \sim q^{-2}$)

Extended polymer chain : *increase* at large q -values (asymptotic behaviour in $I(q) \sim q^{-1.x}$)

Dimensionless Kratky Plots of unfolded proteins



The bell shape vanishes as folded domains disappear and flexibility increases.

The curve increases at large q as the structure extends.

Hypothesis : the particle has a well-defined interface with the surrounding buffer and a uniform electronic density

$$Q = \int_0^{\infty} I(q) \cdot q^2 dq = 2\pi^2 \phi \cdot (1-\phi) \cdot r_e^2 (\Delta\rho)^2$$

Porod invariant

$$\phi = \frac{N \cdot V_{\text{obj}}}{V}$$

Protein volumic concentration

Number of proteins

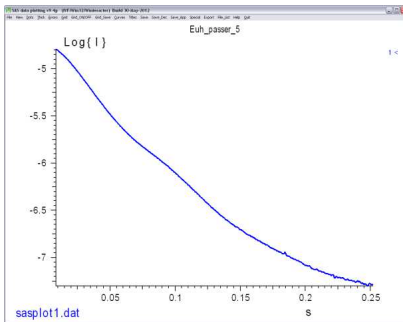
Protein volume

Solution volume

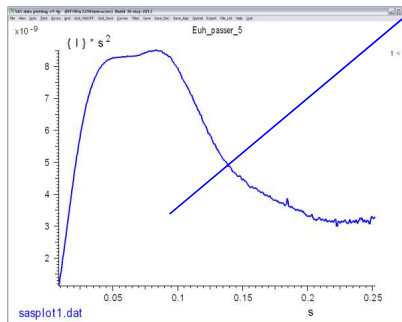
Does not depend on shape, only on contrast

The Porod Invariant is the integral of this curve

Representation : Log I(Q) vs Q



Kratky representation : I(Q)·Q² vs Q



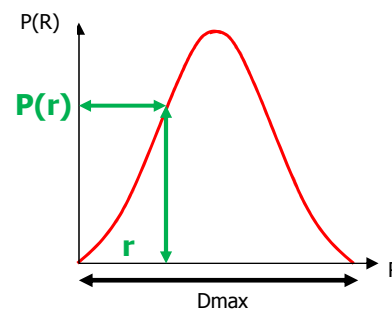
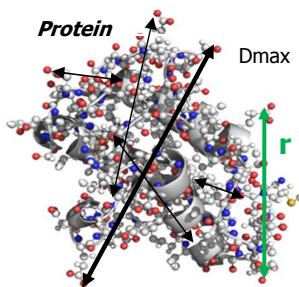
$$Q = \int_0^{\infty} I(q) \cdot q^2 dq$$

$$V_{\text{obj}} = \frac{2\pi^2 \cdot I(0)}{Q}$$

- Valid for diluted systems
- Does not require absolute units
- Not valid for unfolded objects
- Not precise

Back to real space : Distance Distribution Function

The distance distribution function represent the distribution of the distance between each atoms pair. The size of the particle is limited and satisfied the conditions where P(r=Dmax) = 0 and P(r=0) = 0



The radius of gyration and the intensity at the origin can be derived from P(r) using the following expressions :

$$R_g^2 = \frac{\int_0^{D_{\text{max}}} r^2 P(r) dr}{2 \int_0^{D_{\text{max}}} P(r) dr}$$

$$I(0) = 4\pi r_e^2 \phi \int_0^D P(r) dr$$

This alternative estimate of R_g makes use of the whole scattering curve, and is much less sensitive to interactions or to the presence of a small fraction of oligomers. Comparison of both estimates : useful cross-check

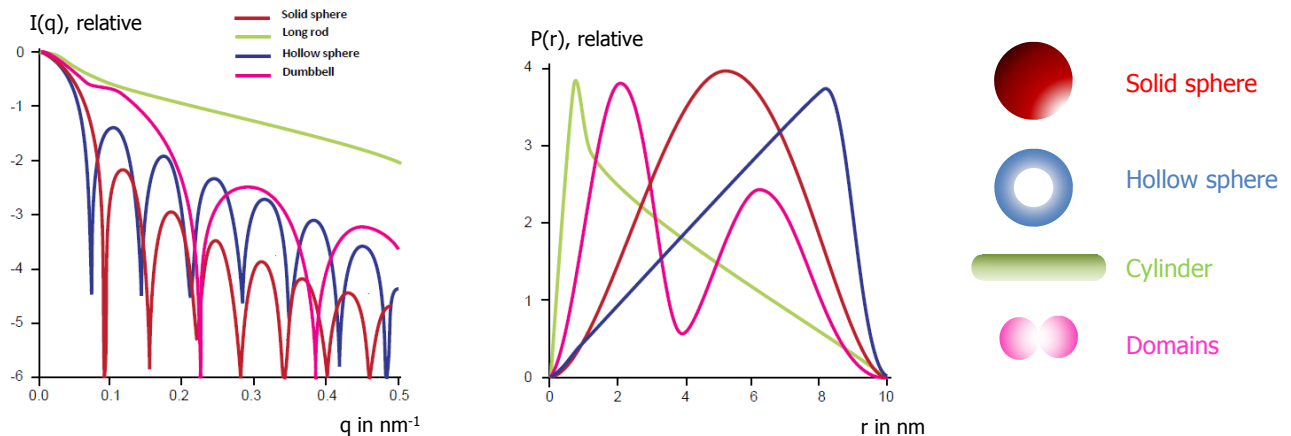
The scattered intensity $I(q)$ can be written with the distribution function $P(r)$. $P(r)$ function is calculated with indirect Fourier transform applied to the scattered intensity $I(q)$. The both curves contain the same information.

$$I(q) = 4\pi r_e^2 \varphi \int_0^{D_{\max}} P(r) \frac{\sin(qr)}{qr} dr$$

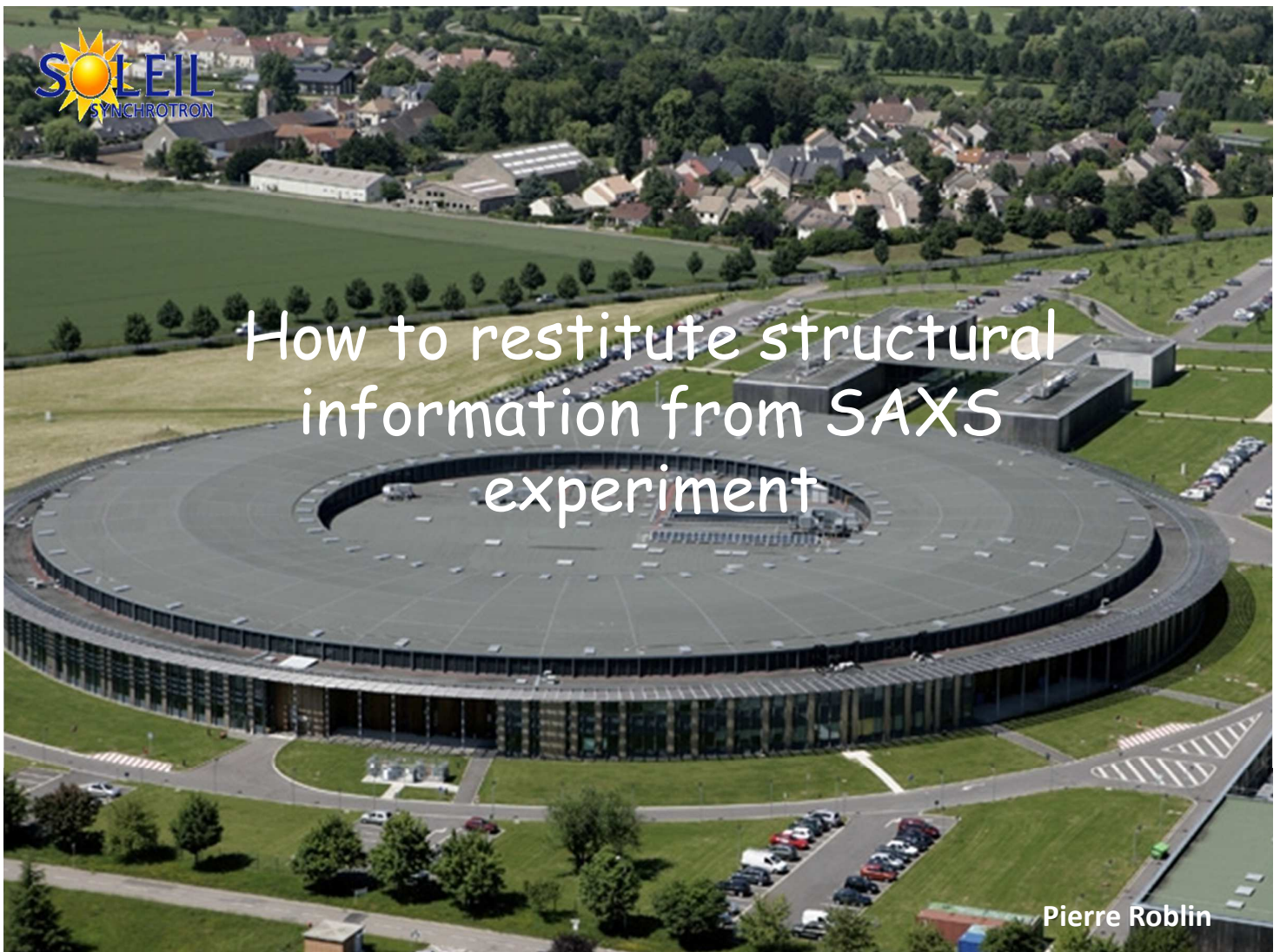
with $D_{\max}$ as maximal distance in the particle

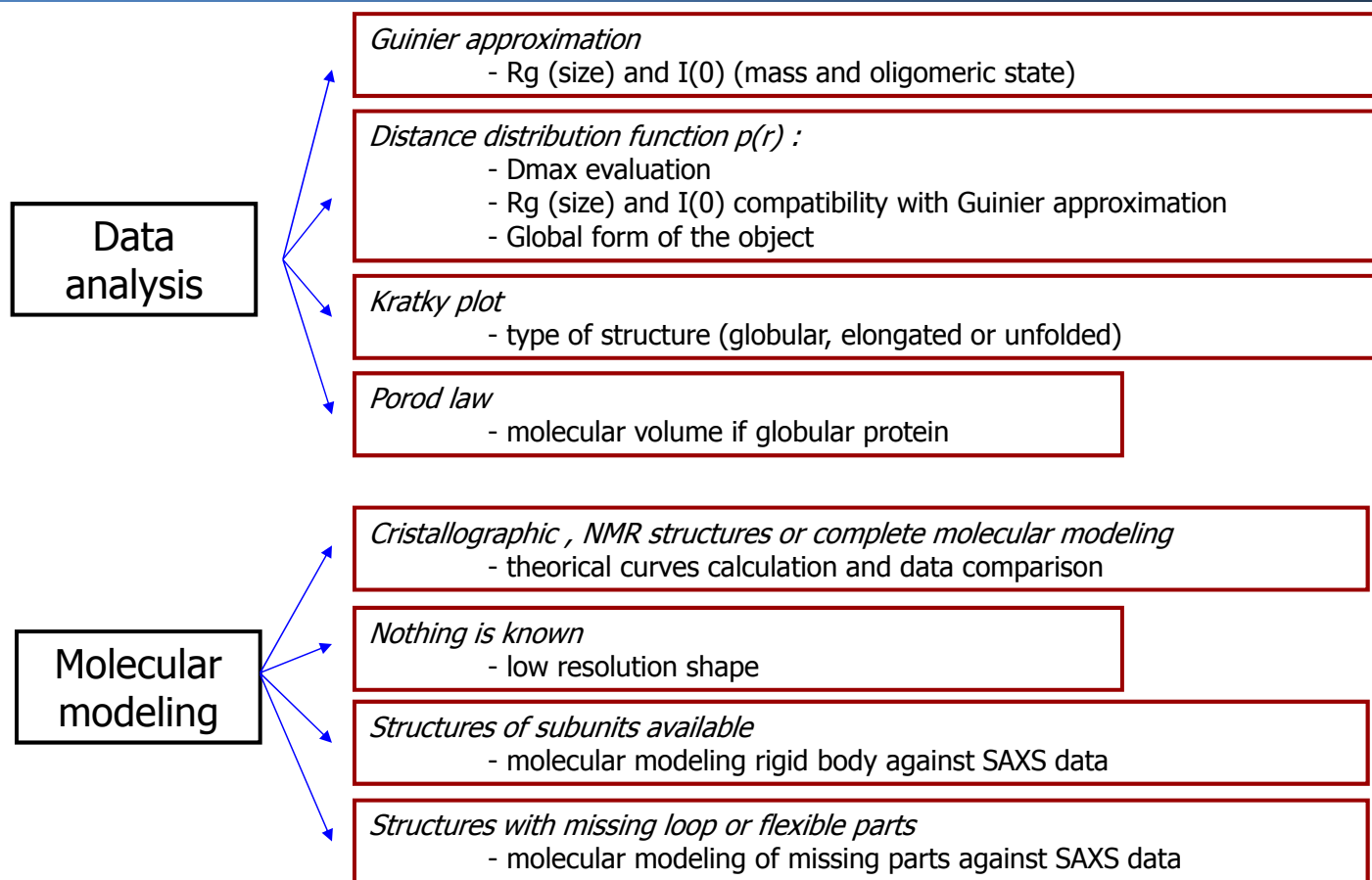
$$P(r) = \frac{r^2}{2\pi^2 \varphi r_e^2} \int_0^\infty q^2 I(q) \frac{\sin(qr)}{qr} dq$$

However, direct calculation of $P(r)$ from $I(q)$ is made difficult and risky by $[q_{\min}, q_{\max}]$ truncation and data noise effects.



The pair distribution function entirely depends on the shape of the particle





From an atomic structure to a solution scattering pattern

The scattering pattern of a particule with an atomic structure resolved by crystallography or NMR can be solved analytically

Debye method to compute scattering of electrons from nuclear position :

$$I(q) = \sum_{i=1}^M \sum_{j=1}^M F_i(q) F_j(q) \frac{\sin(q \cdot r_{i,j})}{q \cdot r_{i,j}}$$

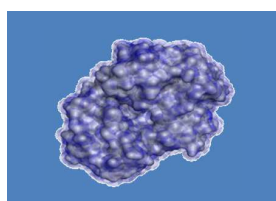
$F_i(q)$, $F_j(q)$, Form factor of atom i and atom j

M number of atom in the protein

Distance r between atom i and atom j

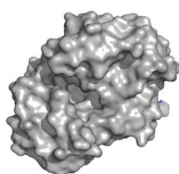
Approach computationally expensive and time-cost increases quadratically with the number of atom in the protein

The experimental scattering curves are obtained by substracting the contribution of the solvent. But the solvated molecules have a border of solvent bound with a diffusion density different from the disordered solvent



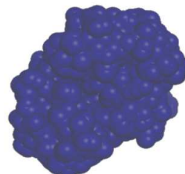
Molecule in solution

=



Molecule in vacuum

+



Hydrated molecule

-



Excluded volume

$$I_{th}(q) = \left\langle \left| A_a(\vec{q}) - \rho_s A_s(\vec{q}) + \delta\rho_b A_b(\vec{q}) \right|^2 \right\rangle_{\Omega}$$

$A_a(q)$ = atomic scattering in vacuum

$A_a(q)$ = scattering from the hydration shell, layer of thickness $3\text{\AA}$

$A_s(q)$ = scattering from excluded volume

In CRY SOL program, in order to gain computing time, $I(q)$ is developed in a series of Bessel functions and spherical harmonics :

$$I_{calc}(q) = \sum_{l=0}^L \sum_{m=-1}^l \left| A_{lm}(q) - \rho_0 C_{lm}(q) + \delta\rho B_{lm}(q) \right|^2$$

The experimental scattering curves are then fitted using only 3 parameters in order to minimize the discrepancy χ :

- the general scale of $I_{calc}(q)$
- the total excluded volume V , which is equivalent to modifying the average contrast
- the contrast of the border layer $\delta\rho$

$$\chi^2 = \frac{1}{N-1} \sum_{i=1}^N \left[\frac{I_{exp}(q_i) - scale * I_{calc}(q_i)}{\sigma_{exp}(q_i)} \right]^2$$

Svergun , Barberato & Koch (1995), *J. Appl. Cryst.*, **28**, 768

Svergun D, Barberato C, and Koch M.H.J. (1995) **CRY SOL** - a program to evaluate x-ray solution scattering of biological macromolecules from atomic coordinates.

J. Appl. Cryst. 28, 768

Most popular for BioSAXS, stand-alone program, fit model to data, fast computational algorithm

<http://www.embl-hamburg.de/biosaxs/atsas-online/crysol.php>

Grishaev A, Guo L, Irving T, Bax A. (2010) **AXES** Improved Fitting of Solution X-ray Scattering Data to Macromolecular Structures and Structural Ensembles by Explicit Water Modeling. *J. Am. Chem. Soc.* 132, 15484-6.

Use explicit water modeling solvation layer, robust fitting approach

<http://spin.niddk.nih.gov/bax/nmrserver/saxs1/>

J. Bardhan, S. Park and L. Makowski (2009) **SoftWAXS**: a computational tool for modeling wide-angle X-ray solution scattering from biomolecules *J. Appl. Cryst.* 42, 932-943

A program to compute WAXS,

Upon request

Schneidman-Duhovny D, Hammel M, Sali A. (2010) **FoXS**: a web server for rapid computation and fitting of SAXS profiles. *Nucleic Acids Res.* 38 Suppl:W540-4.

Debye-like computation, web server based

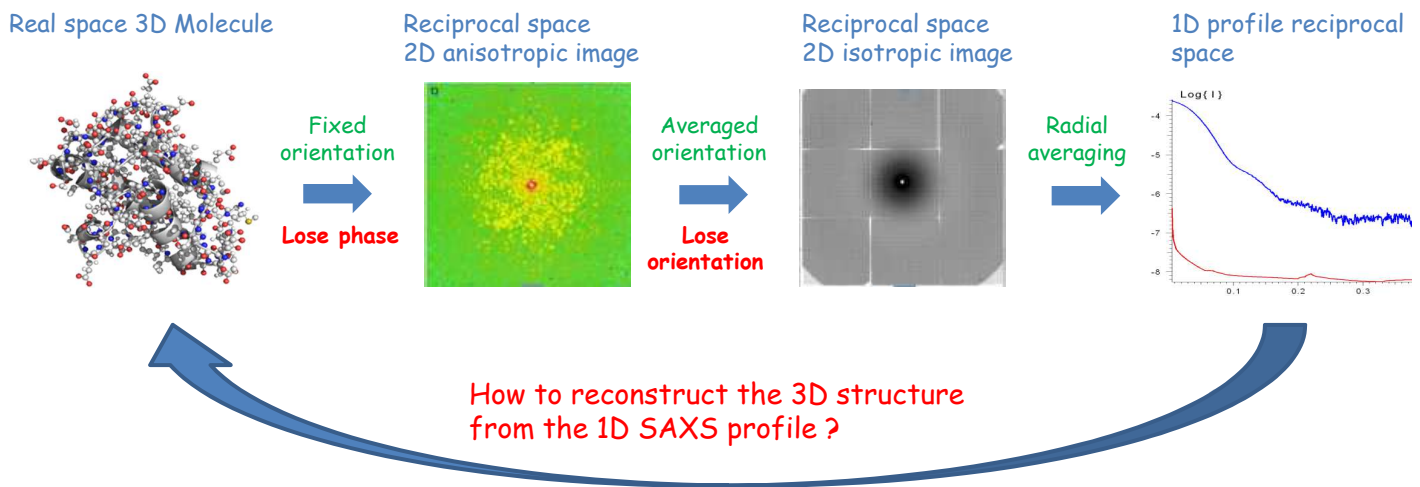
<http://modbase.compbio.ucsf.edu/foxs/>

Zuo X, Zhang R, Tiede DM. **SolIX**: A computer program for solution molecular x-ray scattering simulations . *Photosynth Res.* 2009 Nov-Dec; 102(2-3): 267-279.

Debye-like computation, Windows-based, can handle non-standard atoms/residues, for biomolecules and supramolecules

Upon request zuox@anl.gov, tiede@anl.gov

The 1D SAXS profile is the Fourier transform of the 3D structure. Contrary to the direct scattering calculation, the inverse problem cannot be solved analytically, i.e., no "inverse Debye" formula can be constructed to yield 3D position coordinates from scattering data.



Ambiguity of the solution :

One 3D structure → One SAXS curve

BUT

One SAXS curve → **Many 3D structures**

3D shape reconstructions from SAXS data with DAMMIN

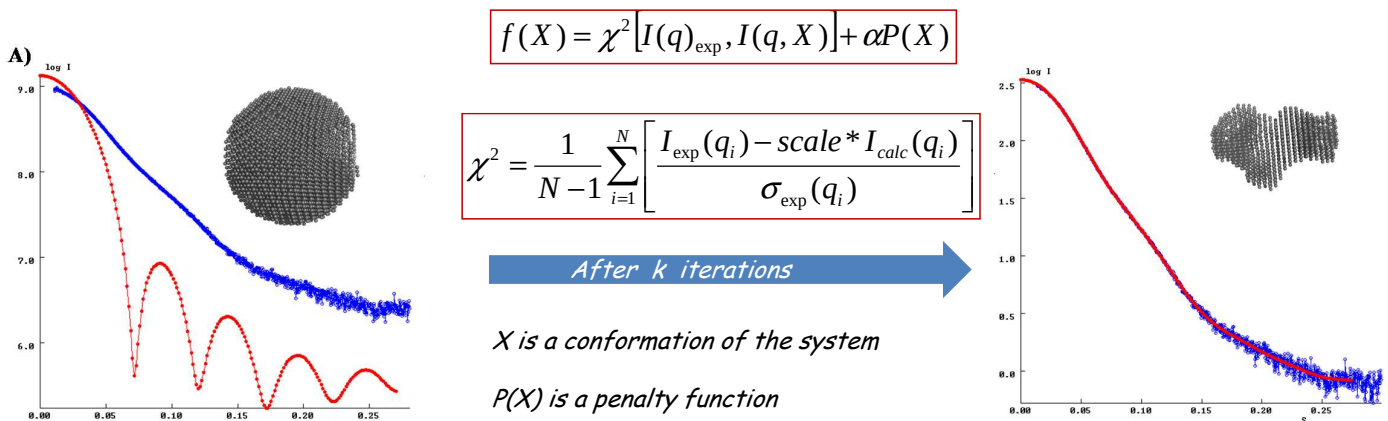
Ab initio shape modelling: nothing is known excepted the curve !

Principle of the method: any structure can be approximated at any resolution by a set of spheres of small enough diameter

Starting model = sphere with a radius $R = D_{\max}/2$ with N scattered beads ($r_0 \ll R$)

The number of the "dummy atom" $N \approx (R/r_0)^3$

Each sphere is associated to a position j and an index X_j corresponding to the type of the phase ($X_j = 0$ for solvent and $X_j = 1$ for molecule)



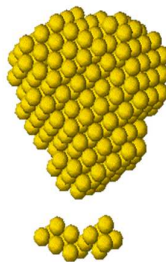
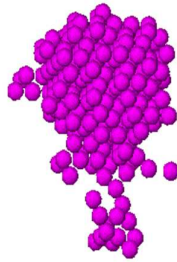
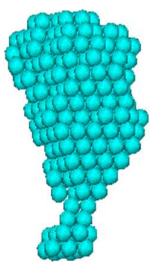
Obtaining 3D shapes from SAXS data is a defined problem that could be solved by introducing additional information to reduce ambiguity of interpretation

Introduction of the penalty function to limit the formation of discontinuous models or disjoint spheres

$$P(X) = 1 - \langle C(N_e) \rangle \quad C(N_e) = 1 - \exp(-0.5N_e)$$

N_e is the number of contact of a sphere with the neighboring spheres, N_e is equal to 12 in hexagonal lattice.

- In this case where the sphere has a maximum contact $C(12) = 1$ so $P(X) = 1 - 1 = 0$ (no penalty)
- With disconnected or loose sphere where $C(0) = 0.002$ so $P(X) = 0.998$ (strong penalty)
- With sphere on the surface $N_e \approx 6$, $C(6) = 0.943$ so $P(X) = 0.057$ (low penalty)



Compact model

Loose model

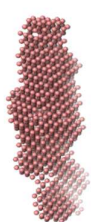
Disconnected model

DAMMIN : necessit to perform a serie of run (20-50) to compare the different shape obtained with the same data.

After the run, an optimal superposition of models is realized with the program suite DAMSEL and DAMSUP.

The algorithm define a criteria of similarity, called « Normalized Spatial Discrepancy » or NSD, which measure the agreement between two models.

For similar shape $NSD < 1$, typically very similar shape $NSD \approx 0.5$



Shp1

Shp2

Shp3

Shp4

Shp5

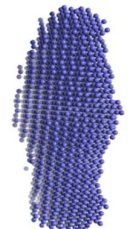
File	Aver	1	2	3	4	5
1	0.52	0.00	0.51	0.52	0.50	0.52
2	0.52	0.51	0.00	0.52	0.49	0.52
3	0.53	0.52	0.52	0.00	0.53	0.52
4	0.53	0.50	0.49	0.53	0.00	0.54
5	0.53	0.52	0.52	0.52	0.54	0.00

Mean value of NSD	:	0.535
Standard deviation of NSD	:	0.008

Damsel.log



Damfild
(average)



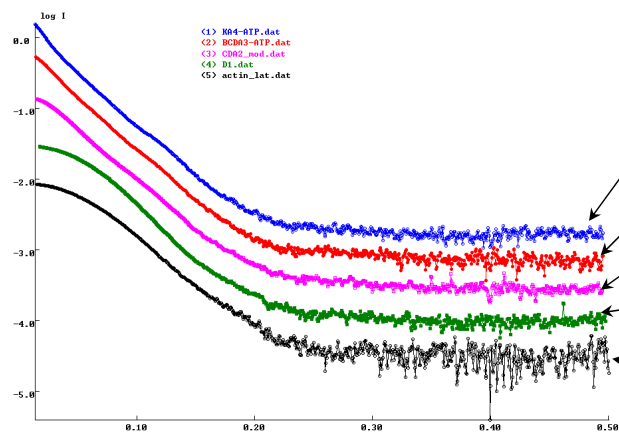
Damaver
(all superimposed)

Model are conserved if the $NSD < \text{Mean of NSD} + 2 \times \text{standart deviation}$

The model with the lowest NSD is the shape which has the most similarities with other, and can be regarded as the most representative of envelopes in accordance with the SAXS data

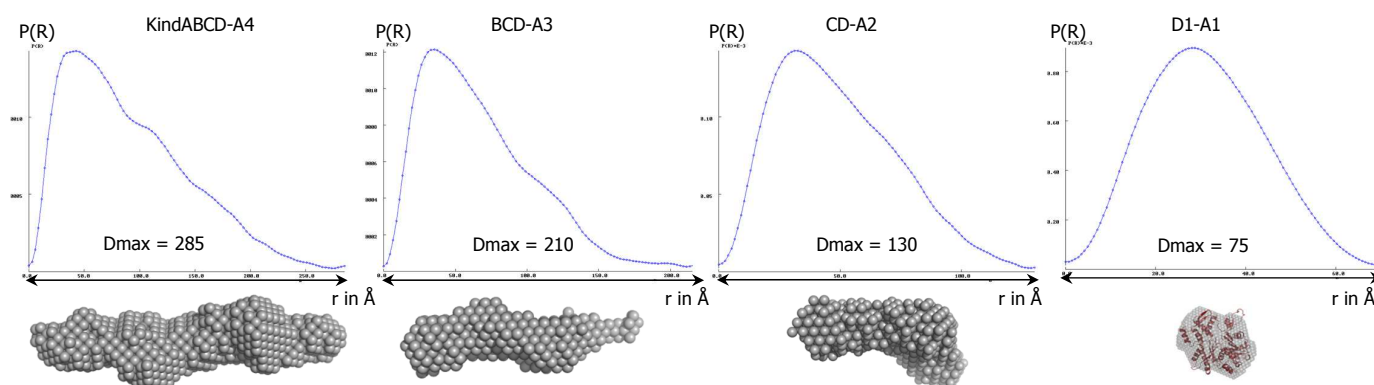
Be careful with damfild.pbd because $I_{\text{damfild}}(q) \neq I_{\text{exp}}(q)$

Scattering curves obtained on different complexes Spire-Actin and Actin alone



Complexes	Radius of gyration	Maximum diameter
	75.5 Å	285 Å
	55.5 Å	210 Å
	38.9 Å	130 Å
	25 Å	75 Å
	23.1 Å	70 Å

Histogram of intramolecular distances and ab initio molecular envelopes determined using DAMMIF

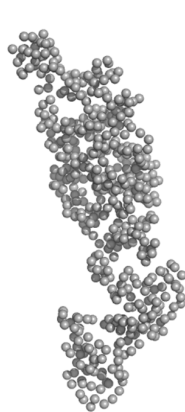
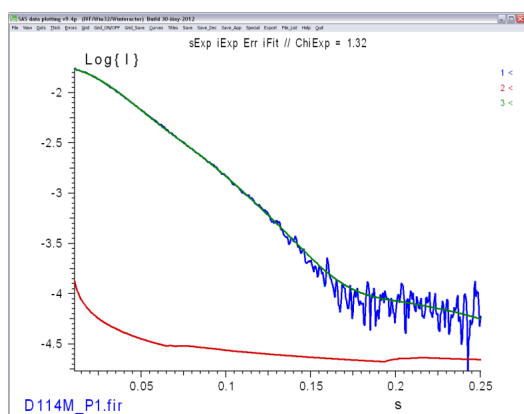


Ab initio model accounting for high resolution data

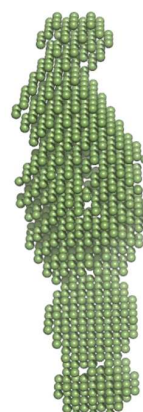
DAMMIN/DAMMIF : very low resolution because restricted portion of the data used ($q < 0.2 \text{ \AA}^{-1}$), and ambiguity of the models

GASBOR : a protein comprising N residues is represented by an ensemble of N spheres centered at the $C\alpha$ positions.

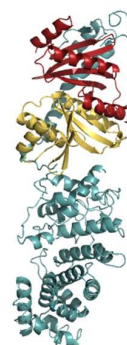
An initial gas-like distribution of dummy residues is refined using Simulated Annealing to fit the data under constraints ensuring a final chain like distribution



GASBOR beads model



DAMMIF shape



High resolution structure

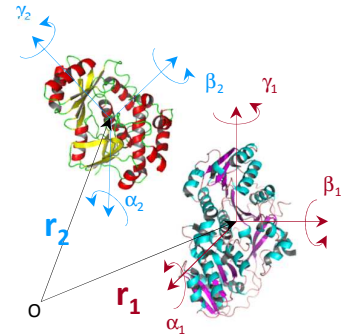
SASREF : when atomic structures of domains are known, but not their mutual organization

The objective is to find the relative orientation of each subunit with a correct agreement with the SAXS data of the complex

The scattering intensity $I(q)$ of the complex is equal to the sum squared of the amplitudes of each subunit

$$I(S) = \left\langle \left| \sum_{k=1}^K A^{(k)}(\vec{S}) \right|^2 \right\rangle_{\Omega}$$

$$A^{(k)}(\vec{S}) = \exp(i\vec{S} \cdot \vec{r}_k) \prod (\alpha_k \cdot \beta_k \cdot \gamma_k) [C^{(k)}(\vec{S})]$$



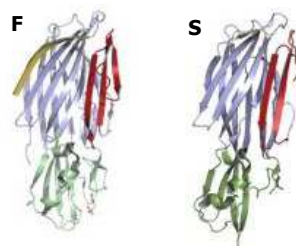
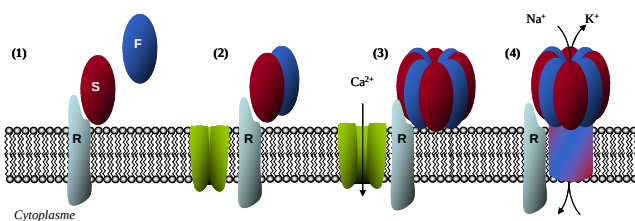
The amplitudes are calculated with CRY SOL from the high resolution structure of each monomer

The algorithm of minimization is the same used with DAMMIN with a penalty function (interconnectivity of the subunits, the steric clashes) and possibility to give information about contacting residues from other experiences.

$$f(X) = \sum_i \chi_i^2 + \alpha_{dist} P_{dist}(X) + \beta_{cross} P_{cross}(X) + \gamma_{cont} P_{cont}(X)$$

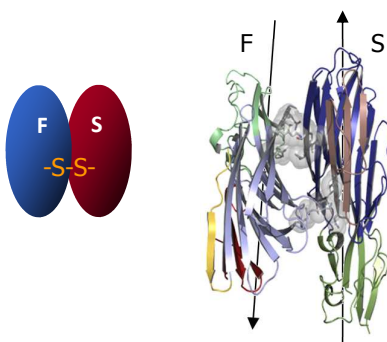
Petoukhov & Svergun (2005). *Biophys. J.*, 89, 1237-1250.

Example : structural characterization of covalent heterodimer

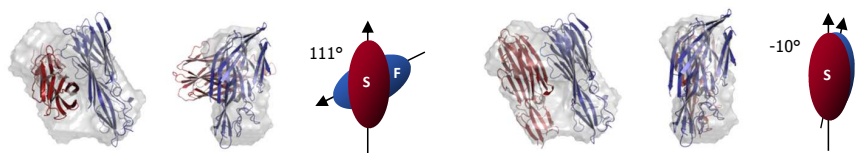
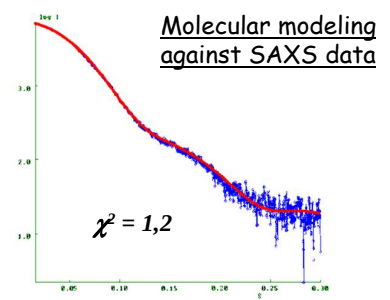
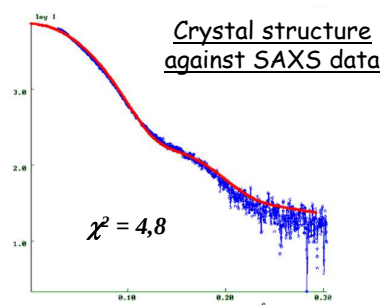


Structure of each subunits solved but no crystal of the hetero-octamer

Conception of a covalent F-S heterodimer with the same biological properties as individual F and S monomers



The structure solved by cristallography is incompatible with the formation of octamer

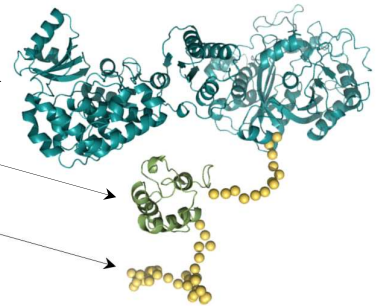


BUNCH and CORAL : quaternary structure analysis of multidomain protein



Combination of rigid body and ab initio modeling :

- position and orientation of rigid domains
- probable conformation of flexible linkers



$$f(X) = \sum_i \chi_i^2 + \alpha_{ang} P_{ang}(X) + \beta_{cross} P_{cross}(X) + \gamma_{dih} P_{dih}(X) + \delta_{ext} P_{ext}$$

As SASREF, the amplitude are calculated with CRY SOL from the high resolution structure of each monomer

The algorithm of minimization is the same used with SASREF with a penalty function including the steric clashes P_{cross} , the dihedral angle P_{ang} and P_{dih} , and the compactness of the loop P_{ext} . The possibility to give information about contacting residues from other experiences is also added.

Flexibility → no unique structure !
NOT a structure but a SAXS data compatible model

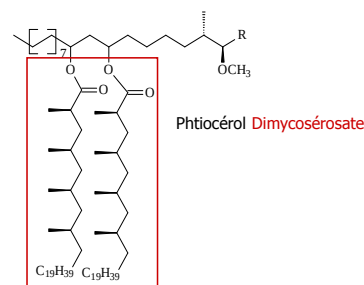
Petoukhov & Svergun (2005). *Biophys. J.*, 89, 1237-1250.

Example : structural characterization of multidomain protein MAS (mycoserosic acid synthase)

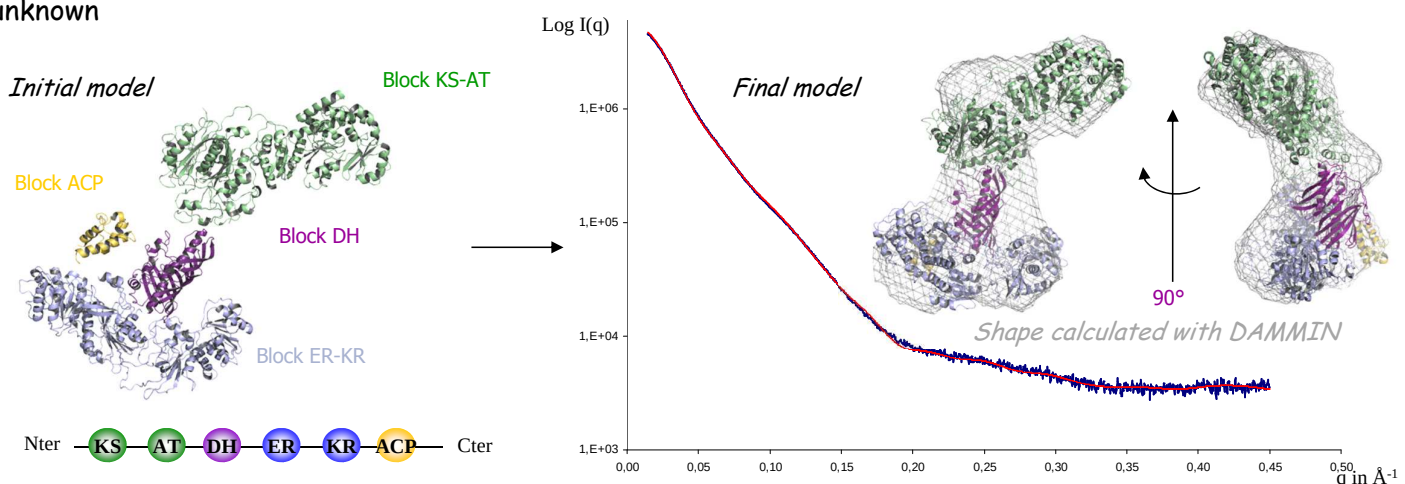


KS : ketosynthase domain
AT : acyl transferase domain
DH : dehydratase domain

ER : enoyl reductase domain
KR : keto reductase domain
ACP : acyl transferase domain



Six catalytic domains, each structure of the domain are known but the structure of entire protein is unknown

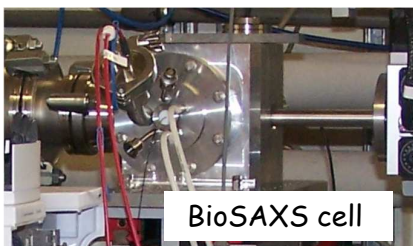
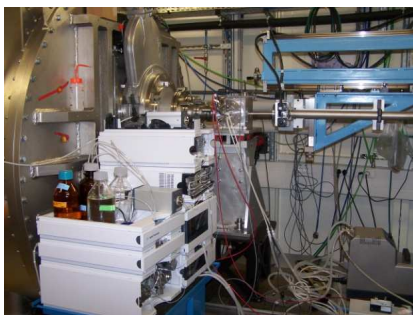


SAXS experiment using HPLC online

Pierre Roblin

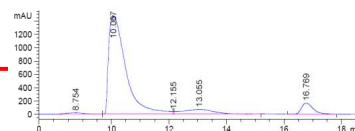
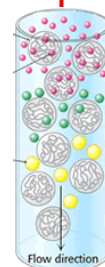
SAXS experiments : why use HPLC-SEC ?

HPLC-SEC coupled to the beamline



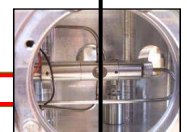
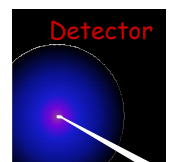
Autosampler

Separation column and
UV detection



Sample for HPLC : 10 μ l (20mg/ml) to 80 μ l for (2,5mg/ml)

Sample for direct injection : 20 μ l to 50 μ l with [protein] > 1mg/ml



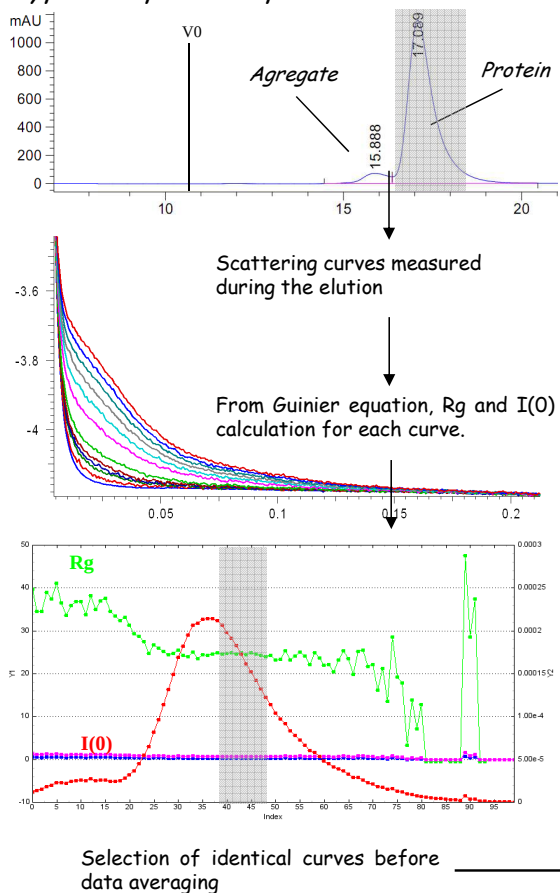
BioSAXS cell

X-rays

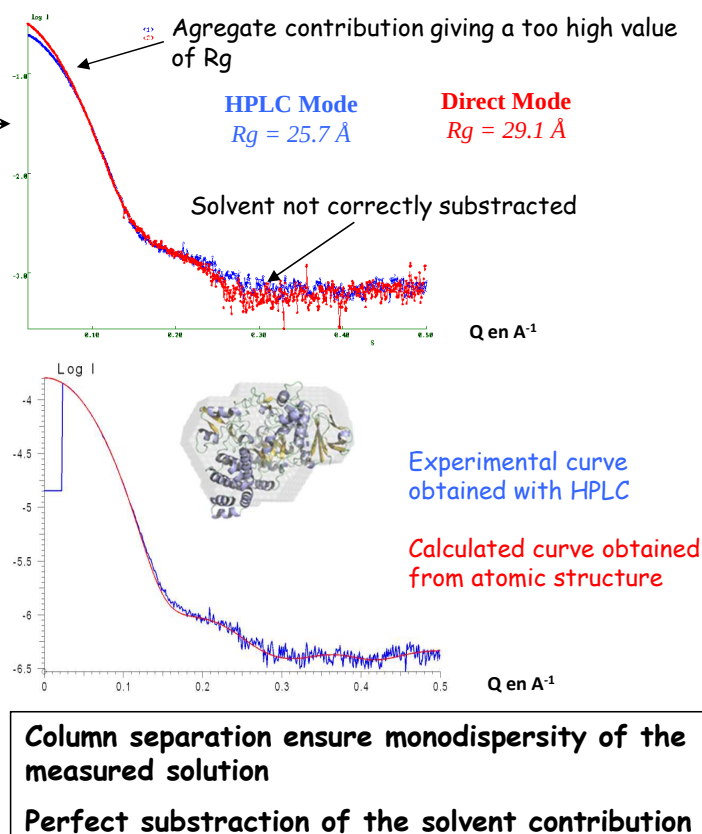
- Monodisperse solution by removing aggregation
- Oligomers separation
- Perfect substraction of the solvent
- Possibility to study protein complex with low Kd

SAXS characterization of macromolecule with coupled HPLC

Typical experience performed with HPLC

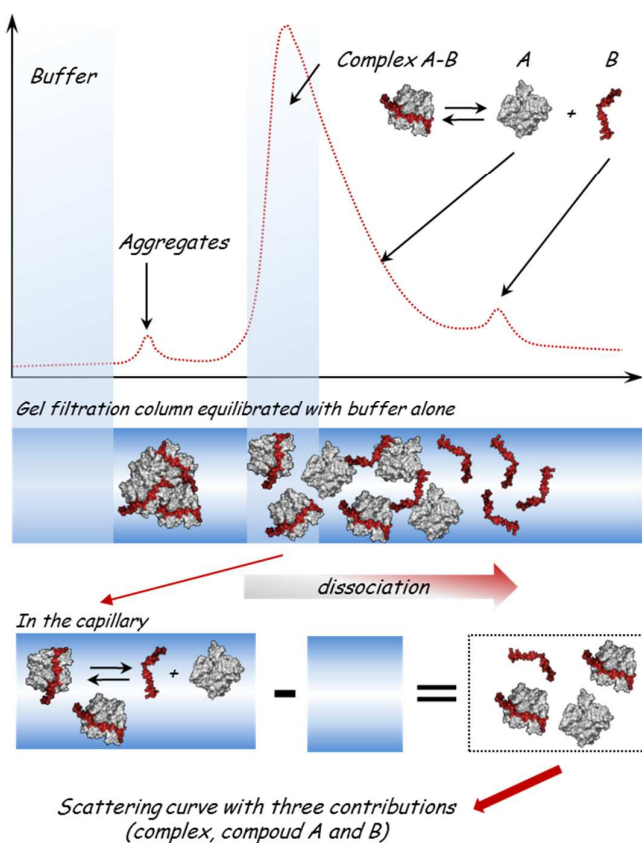


Comparison between curve obtained with direct injection and after HPLC method

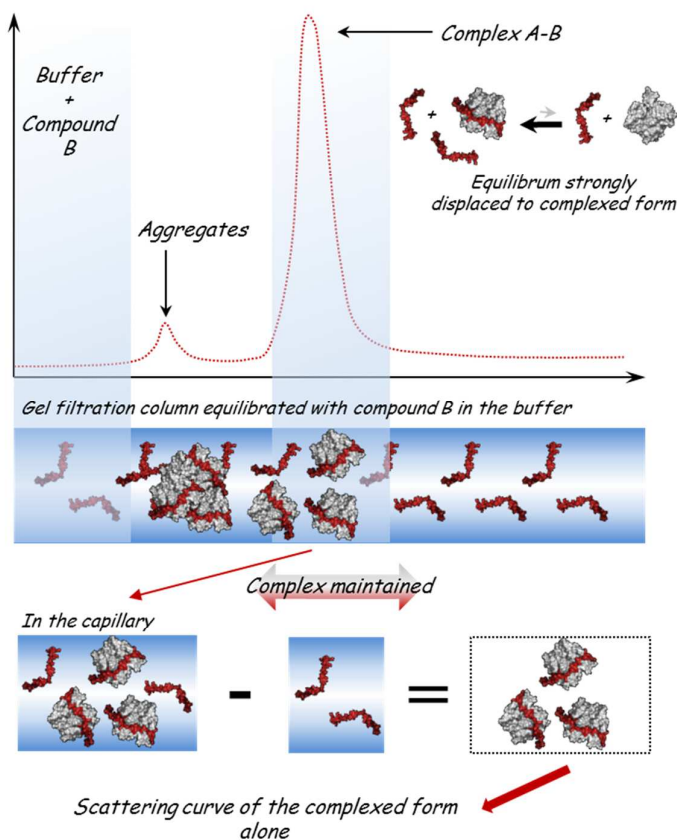


How to deal with the complex with low affinity

HPLC with no compound in the buffer



HPLC with compound in the buffer



To perform SAXS experiment don't forget :

- analysis and modeling require a monodisperse and ideal solution (~50 to 100 μ l up to 2mg/ml)
 - structure of the protein or homolog , the subunits or different parts of the complex must be known
 - missing parts (internal loop or N or C-ter part) represent less than 10% of the mass
- because of the ambiguity of the solution, don't forget to validate your model with other technics that can constrain the field of possibilities (EM,NMR, mutagenesis...)

IN



SAXS



OUT

